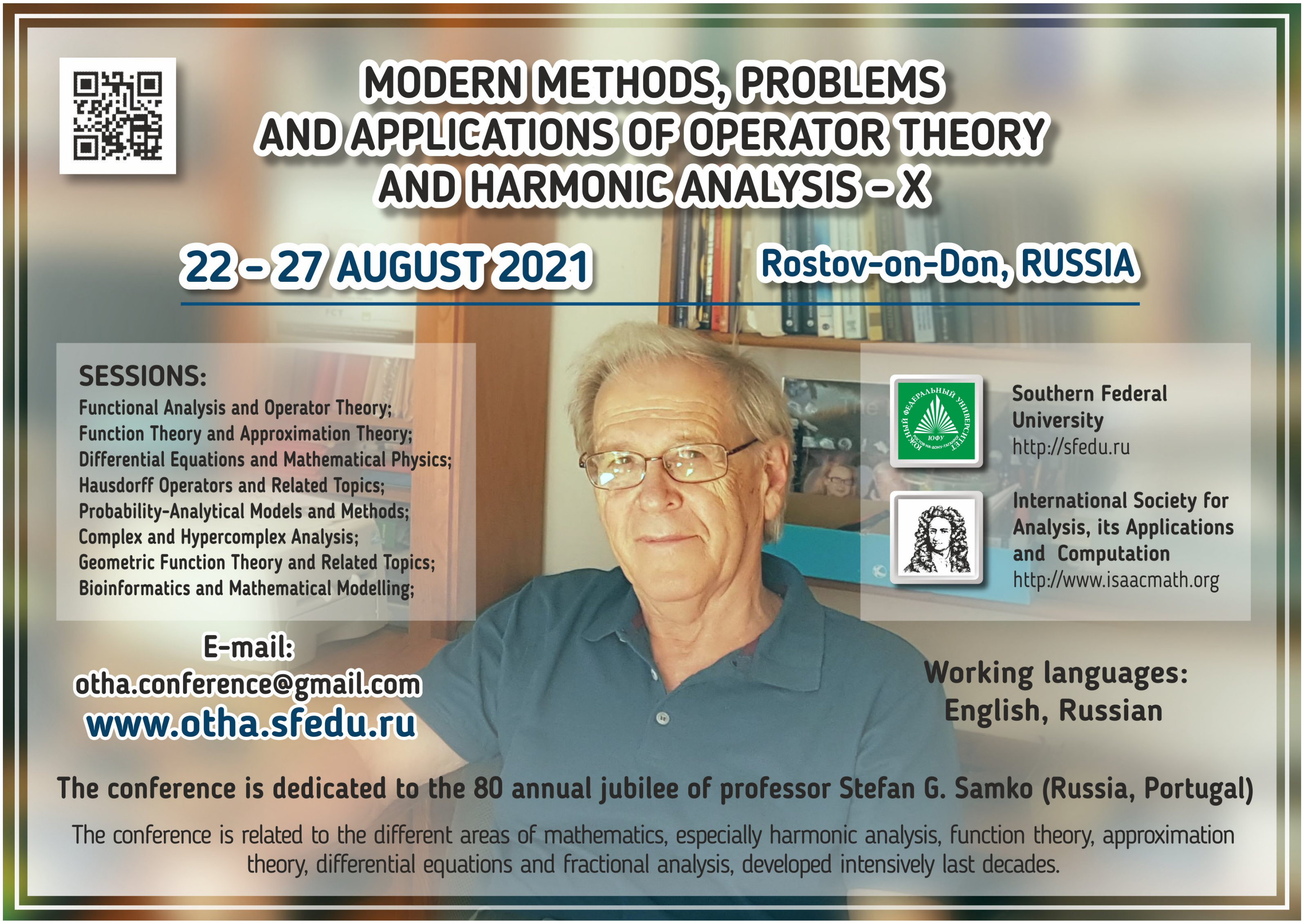

BOOK OF ABSTRACTS

Tenth International Scientific Conference

"Modern Methods, Problems and Applications of Operator Theory and Harmonic Analysis —X"

OTHA-2021

Rostov-on-Don, 2021.

www.otha.sfedu.ru/conf2021/

E-mail: otha.conference@gmail.com

Rostov-on-Don
2021

Table of content

Analysis

Differential equations and mathematical physics

Probability-Analytical Models and Methods

E. V. Alymova (Rostov-on-Don, Russia)
langnbsp@gmail.com

FORECASTING AN EFFECTIVE TRADING STRATEGY USING A PROGRAM MODEL BASED ON MACHINE LEARNING METHODS

The purpose of the study is to develop a methodology for a reasonable choice of a trading strategy on the currency exchange in the next ten-minute period, based on the data of the current twenty-minute trading period.

This research continues the study [1], in which a model for classifying the behavior of a financial time series based on indicators of the logarithmic return of the BTC / USD currency pair implements in the form of a neural network.

The study assumes that the logarithmic return indicators during the current twenty-minute period determine the logarithmic return indicators in the next ten-minute period.

To prove this assumption, we build a model for predicting the value of the linear regression coefficient of logarithmic returns in the next ten-minute period based on the known indicators of logarithmic returns in the previous twenty-minute period.

The proposed approach identifies three trading activities: buying at the beginning of the next period to sell at the first higher price (strategy 1), sell at the beginning of the next period to buy at the first lower price (strategy -1). The third strategy is inaction (strategy 0), when the linear regression coefficient's value at the beginning of the next ten-minute period is insignificant for the chosen criterion.

As the experiments on historical data show, every sixth decision in choosing a strategy leads to losses without introducing strategy 0, and when strategy 0 is applying, every eighth decision is unprofitable.

The feed-forward neural network is using to predict future values of the linear regression coefficients. Neural network parameters were selected empirically to improve forecasting accuracy.

This work was supported by the Russian Foundation for Basic Research under Grant No 18-01-00910.

REFERENCES

1. *Alymova E. V., Kudryavtsev O. E.* Neural networks usage for financial time series prediction. Abstracts of Talks Given at the 4th International Conference on Stochastic Methods. Theory of Probability and Its Applications. 2020. Vol. 65, No. 1, pp. 122–123

T. M. Andreeva (Rostov-on-Don, Russia)
metzi@yandex.ru

THE SURJECTIVITY OF CONVOLUTION OPERATORS AND THE SOLVABILITY OF NON-HOMOGENEOUS CONVOLUTION EQUATIONS ON HOLOMORPHIC WEIGHTED SPACES IN BOUNDED CONVEX DOMAINS

Let G be a domain in $\mathbb{C}$ and $H(G)$ the space of all holomorphic functions in G. For a continuous function (a weight) $v : G \to \mathbb{R}$ define the Banach space

$$H_v(G) := \Big\{ f \in H(G) : ||f||_v := \sup_{z\in G} |f(z)|e^{-v(z)} < \infty \Big\}.$$

For an increasing sequence of weights $V = (v_n)$ define the inductive limit

$$\mathcal{V}H(G) := \mathrm{ind} H_{v_n}(G).$$

Let μ be an analytic functional on $\mathbb{C}$ carried by a convex compact set K. With some restrictions on weight sequence which are equal to those used by V.V. Napalkov [1] the continuity and surjectivity problem is studied of the convolution operator

$$\mu * f(z) : f \mapsto \mu_w f(z+w)$$

that maps $\mathcal{V}H(G+K)$ into (onto) $\mathcal{V}H(G)$. The aim of this talk is to establish the surjectivity criteria for convolution operator in terms of its Laplace (Fourier-Borel) transform $\hat{\mu}(\zeta) := \mu_z e^{\langle z \cdot \rangle}$ via the appropriate description of functional weighted spaces that are conjugated to $\mathcal{V}H(G+K)$ and $\mathcal{V}H(G)$. Similar research was presented in [2] for the spaces of functions that are holomorphic in convex domains and have a polynomial growth near the boundary (the weight sequence $v_n(z) = n\ln(1+|z|)$).

Under some restrictions on $\hat{\mu}$ the results on the solvability of the non-homogeneous convolution equation

$$\mu * f(z) = h(z),$$

where $f \in \mathcal{V}H(G+K)$, $h \in \mathcal{V}H(G)$, are presented in the case when the symbol $\hat{\mu}$ has zeroes on G. Similar research was presented in [3] for the spaces of functions that are holomorphic in Runge domains in $\mathbb{C}^n$.

REFERENCES

1. *Napalkov V. V.* Spaces of analytic functions of prescribed growth near the boundary. Mathematics of the USSR-Izvestiya. 1988. Vol. 30, No. 2. P. 263.

2. *Abanin A. V., Ishimura R., Khoi L. H.* Convolution operators in $A^{-\infty}$ for convex domains. Arkiv för matematik. 2012. Vol. 50. —№ 1. —P. 1–22.

3. *Trutnev V. M.* Convolution Equations in spaces of Entire Functions of Exponential Type. Journal of Mathematical Sciences 120, 1901–1915 (2004).

A. D. Bendikov
(Mathematical Institute, Wroclaw University, Wroclaw, Poland)
bendikov@math.uni.wroc.pl

ON THE SPECTRUM OF SCHRÖDINGER-TYPE OPERATORS: THE CASE OF POTENTIALS WITH LOCAL SINGULARITIES

The goal of this talk is twofold. We prove that the operator $H = L + V$, the perturbation of the Taibleson-Vladimirov multiplier $L = \mathfrak{D}^{\alpha}$ by the potential

$$V(x) = b\,\|x\|^{-\alpha}, \quad b \geq b_{*},$$

is essentially self-adjoint and non-negative definite (the critical value b_{*} depends on α and will be specified in our talk). While the operator H is non-negative definite the potential $V(x)$ may well take negative values, e.g. $b_{*} < 0$ for all $0 < \alpha < 1$. The equation $Hu = v$ admiits a Green function $G_H(x, y)$, the integral kernel of the operator H^{-1}. We obtain sharp lower- and upper- bounds on the ratio of the functions $G_H(x, y)$ and $G_L(x, y)$. Examples illustrate our exposition.

N. V. Boev (Rostov-on-Don, Russia)
nvboev@sfedu.ru

EXPLICIT EXPRESSIONS FOR DISPLACEMENTS IN BACK-REFLECTED SHORT WAVES FROM 2D AND 3D DEFECTS IN ELASTIC MATERIALS

The explicit form of the main term of the asymptotic behavior of the displacement amplitude is written out in the case of backscattering of the longitudinal and transverse waves in the far-field approximation. The applied significance of this result lies in the fact that when detecting obstacles in acoustic media and defects in elastic media with ultrasonic non-destructive testing, sounding by high-frequency acoustic and elastic waves in the echo mode is used. This type of scanning allows you to get in any direction the transit time of the reflected echo signal and its amplitude. Such data form the basis of the method for reconstructing complex obstacles.

To sound an elastic medium with possible defects, a time-finite pulse with tonal filling with a sufficient number of wavelengths (for example, 6 - 10 waves) of harmonic oscillations is used. This allows us to consider the problem in the approximation of a model of steady vibrations of an elastic medium. In this case, we assume that the defect is located at a sufficient distance from the oscillation source, which makes it possible to use the far-field approximation at high oscillation frequencies in calculating the waves incident and reflected from the defect surface.

An ultrasonic longitudinal plane wave introduced into the elastic material interacts with the surface of the obstacle, is scattered on it, and the sensor records the backward wave from the defect. If the surface of the scatterer is smooth, then the first arriving pulse will be reflected from the vicinity of a point on the surface, the normal in which is parallel to the direction of propagation of the high-frequency radiated longitudinal wave. By replacing a plane wave introduced into the material with a superposition of point sources of spherical waves, each of which is caused by a concentrated force that changes in time according to a harmonic law, the original problem is reduced to studying the problem in a local setting. The study of the local problem was carried out on the basis of the physical theory of Kirchhoff diffraction.

By the methods of the geometric theory of diffraction in the far-field approximation, explicit expressions for the displacements for back reflected longitudinal and transverse waves from the surfaces of 2D and 3D obstacles are obtained.

N. P. Bondarenko (Saratov, Samara, Russia)
bondarenkonp@info.sgu.ru

AN INVERSE SPECTRAL PROBLEM FOR A FUNCTIONAL-DIFFERENTIAL OPERATOR WITH INVOLUTION

The aim of the talk is to propose an approach to inverse spectral problems for functional-differential operators with involution. For definiteness, we focus on the following second-order functional-differential equation with involution-reflection:

$$-\alpha u''(x) - u''(-x) + p(x)u(x) + q(x)u(-x) = \lambda u(x), \quad x \in (-1,1), \tag{1}$$

where λ is the spectral parameter, $\alpha \in (-1,1) \cup (\mathbb{C} \setminus \mathbb{R})$, $p, q \in L_1(-1,1)$. It is proved that the functions $p(x)$ and $q(x)$ are uniquely specified by the five spectra of the boundary value problems for equation (1) with the following regular boundary conditions:

$$(i)\ u(-1) = u(1) = 0, \quad (ii)\ u'(-1) = u(1) = 0,$$
$$(iii)\ u(-1) = u'(-1) = 0, \quad (iv)\ u(1) = u'(1) = 0,$$
$$(v)\ u(-1) = u'(1) = 0.$$

Our method is based on the reduction of equation (1) to the matrix form

$$-Y'' + Q(x)Y = \lambda W Y,$$

where $Y(x)$ is a two-element vector function, $Q(x)$ is a (2×2) matrix function, $W = \operatorname{diag}\{w_1, w_2\}$ is the constant weight matrix, $w_1 w_2 \ne 0$, $\arg w_1 \ne \arg w_2$. The proofs are presented in [1].

Acknowledgement. This work was supported by Grant 20-31-70005 of the Russian Foundation for Basic Research.

REFERENCES

1. *Bondarenko N. P.* Inverse spectral problems for functional-differential operators with involution. ArXiv:2107.12209 [math.SP].

S. A. Buterin (Saratov, Russia)
buterinsa@info.sgu.ru

UNIFORM STABILITY OF SINE-TYPE FUNCTIONS WITH ASYMPTOTICALLY SEPARATED ZEROS

Fix $N \in \mathbb{N} \cup \{0\}$ and consider an entire function of the form

$$\theta(z) = P_N(z)S(z) + \int_{-b}^{b} w(x)\exp(izx)\,dx, \quad w(x) \in L_2(-b,b), \tag{1}$$

where $S(z)$ is a sine-type function (see, e.g., [1]) of the exponential type b whose zeros $\{z_n^0\}_{n\ge1}$ obey the condition $\inf|z_n^0 - z_k^0| > 0$ for $n \neq k$ and $n,k \gg 1$, while $P_N(z)$ is a polynomial of degree N.

In a standard way, using Rouché's theorem, one can show that the zeros $\{z_n\}_{n\ge1-N}$ of $\theta(z)$ have the form

$$z_n = z_n^0 + \frac{\varkappa_n}{\mu_n^N}, \quad \{\varkappa_n\} \in l_2, \quad \mu_n := \left\{ \begin{array}{ll} z_n^0, & z_n^0 \neq 0, \\ 1, & z_n^0 = 0, \end{array} \right.$$

where $\{z_n^0\}_{n=\overline{1-N,0}}$ are zeros of $P_N(z)$. Consider also another function $\tilde\theta(z)$ of the form (1) with a different $\tilde w(x) \in L_2(-b,b)$ but with the same $P_N(z)$ and $S(z)$. Let $\{\tilde z_n\}_{n\ge1-N}$ be zeros of $\tilde\theta(z)$.

Theorem 1. *For any $r > 0$, there exists C_r such that*

$$\|\theta - \tilde\theta\|_{L_2(-\infty,\infty)} = \sqrt{2\pi}\|w - \tilde w\|_{L_2(-b,b)} \le C_r \|\{\mu_n^N(z_n - \tilde z_n)\}_{n\ge1-N}\|_{l_2}$$

as soon as $\|\{\mu_n^N(z_n - z_n^0)\}_{n\ge1-N}\|_{l_2} \le r$, $\|\{\mu_n^N(\tilde z_n - z_n^0)\}_{n\ge1-N}\|_{l_2} \le r$.

Theorem 1 unifies similar facts for the characteristic functions of Sturm–Liouville-type operators [2, 3]. To the form (1), one can convert also the characteristic determinants of (strongly) regular [4] first- and second-order differential operators and pencils, as well as Dirac and matrix Sturm–Liouville operators along with operators on graphs all possessing asymptotically separated spectra, and their perturbations.

This work was supported by Grant 20-31-70005 of the RFBR.

REFERENCES

1. *Levin B. Ja. and Ljubarskiĭ Ju. I.* Interpolation by means of special classes of entire functions and related expansions in series of exponentials. Math. USSR Izvestija. 1975. Vol. 9, No. 3, pp. 621–662.
2. *Buterin S.* Uniform stability of the inverse spectral problem for a convolution integro-differential operator. Appl. Math. Comp. 2021. Vol. 390, 125592.
3. *Buterin S.* Uniform full stability of recovering convolutional perturbation of the St.–L. op-r from the spectrum. J. Diff. Eqns. 2021. Vol. 282, pp. 67–103.
4. *Naimark M. A.* Linear Differential Operators. Part I. Ungar. 1967.

E. G. Chub, V. A. Pogorelov (DSTU, Rostov-on-Don, Russia)
elenachub111@ gmail.com

ON SOME SOLUTION OF THE PROBLEM OF ESTIMATION OF A TELECOMMUNICATION SYSTEM UNDER CONDITIONS OF PARAMETRIC UNCERTAINTY OF THE STOCHASTIC MODEL OF ITS STATE VECTOR

The aim of the research is to synthesize an algorithm for joint estimation of the state vector of a telecommunication system and identification of the parameters of its state vector in real time under conditions of interference of various physical nature.

The telecommunication system is described by a stochastic model in the form "object-observer". The unknown parameters of its state vector are determined from the condition of the minimum of the functional characterizing the quality of the functioning of the telecommunication system [1]. At the first stage, the posterior probability density of this process is approximated by a system of posterior moments. The assumption made below about the possibility of approximating the probability density by the class of Pearson distributions makes it possible to obtain a closed system of moment equations [2,3]. At the next stage, the application of the maximum principle makes it possible to go to the solution of the two-point boundary value problem. At the last stage, the synthesis technique for an approximate solution of a two-point boundary value problem based on the invariant immersion method [1] allows one to form an approximate value of the state vector as a solution to a system of ordinary differential equations.

The implementation of the proposed algorithm does not impose additional requirements on the computer, which makes it possible to widely use this algorithm in modern telecommunication systems.

This research is supported by the RFBR (project 19-01-00451).

REFERENCES

1. *Hutorcev V. V., Sokolov S. V., Shevchuk P. S* . Contemporary control and filtering principles for stochastic systems (M., Radio i svjaz') 2001, 808 p.

2. *Mit'kin, A.S., Pogorelov, V.A., Chub, E.G.* Using the Pearson Distribu- tion for Synthesis of the Suboptimal Algorithms for Filtering Multi-Dimensional Markov Processes. Radiophysics and Quantum Electronics. 2015. 58(3). 10.1007/ s11141-015-9596-z.

3. *Sinicyn I., Kalmann N., Pugachev N.* Filters (Moscow: Publ Logos) 2006, 640 p.

N. V. Danilova (Rostov-on-Don, Russia)
nvdanilova@sfedu.ru
G. I. Beliavsky (Rostov-on-Don, Russia)
gbelyavski@sfedu.ru
A. D. Logunov (Rostov-on-Don, Russia)
alogunov@sfedu.ru

OPTIMAL PORTFOLIO AND THE CONFIDENCE SET

The portfolio quality be determined by the portfolio return - $r_n = (x, r)$. Vector x is a portfolio that satisfies the condition: $(I, x) = 1$. Components of the vector r are returns of risky assets. The vector of returns is a random vector defined on the probability space $\langle \Omega, F, P \rangle$. Suppose that the vector of returns belongs to some set D with a given probability: $P(D) \geq \eta$. Let's call the set D a confidence set.

The maxmin problem of choosing the optimal portfolio is as follows: $\max\limits_{(I,x)=1} \min\limits_{r \in D} (r, x)$. Let the maxmin problem have a solution $d^* = \max\limits_{(I,x)=1} \min\limits_{r \in D} (r, x)$, then we can guarantee that the portfolio return satisfies $P(r_n < d^*) < 1 - \eta$.

Let the sample of values of the vector of returns $V = \{r_1, r_2, ..., r_n\}$ be defined. Let us choose an ellipsoid as the confidence set and try to choose from all ellipsoids D, satisfying the condition $P(D) \geq \eta$, an ellipsoid of minimum volume. To solve this problem, one can easily choose the minimum integer k for which the inequality holds: $\dfrac{k}{n} \geq \eta$. It is natural to regard the frequency $\dfrac{k}{n}$ as a good estimate of the probability $P(D)$. After choosing k, the problem of choosing an ellipsoid is as follows. For each of the subsets consisting of k elements, solve the problem of calculating the minimum volume ellipsoid containing the elements of the subset. Next, select the subset for which the corresponding ellipsoid will have the smallest volume.

The problem is of combinatorial complexity. A method for calculating an acceptable solution to this problem with significantly less computational complexity is proposed.

If the confidence ellipsoid $(C^{-1}(r - m), r - m) \leq 1$ is chosen, then the problem of choosing the optimal portfolio is as follows:

$$\max_{(I,x)=1} [(x, m) - \sqrt{(Cx, x)}].$$

In the talk we present the results of calculations on real data.

D. Dauitbek (Almaty, Kazakhstan)
dauitbek@math.kz

EXTREME POINTS OF THE SET OF ELEMENTS MAJORISED BY AN INTEGRABLE FUNCTION

Let f be an arbitrary integrable function on a finite measure space (X, Σ, ν). We characterise the extreme points of the set $\Omega(f)$ of all measurable functions on (X, Σ, ν) majorised by f, providing a complete answer to a problem raised by W.A.J. Luxemburg in 1967 [2, Problem 1]. Moreover, we obtain a noncommutative version of this result.

Theorem 1 below is the main result of the present paper, which unifies Ryff's theorem [3, 4] and the classic result for vectors [1] with significant extension. The following theorem yields the complete resolution of Luxemburg's problem in the general setting.

Theorem 1. *Assume that $\mathcal{M}$ is a von Neumann algebra equipped with a faithful normal tracial state τ. Let $y \in L_1(\mathcal{M}, \tau)_h$ and let $\Omega(y)$ be defined as the set of all self-adjoint operators $x \in L_1(\mathcal{M}, \tau)$ satisfying $\lambda(x) \prec \lambda(y)$. Then, x is an extreme point if and only if for each $t \in (0, 1)$, one of the following options holds:*

(1). $\lambda(t; x) = \lambda(t; y)$;

(2). $\lambda(t; x) \neq \lambda(t; y)$ *with the spectral projection* $E^x\{\lambda(t; x)\}$ *being an atom in* $\mathcal{M}$ *and*

$$\int_{\{s;\lambda(s;x)=\lambda(t;x)\}} \lambda(s; y) ds = \lambda(t; x)\tau(E^x(\{\lambda(t; x)\})).$$

R E F E R E N C E S

1. *Hardy G., Littlewood J., Pólya G.* Inequalities. Cambridge Univ. Press, Cambridge, 1952.
2. *Luxemburg W. A. J.* Rearrangement-invariant Banach function spaces. Proc. Symp. Analysis, Queen's univ. 1967. pp. 83–144.
3. *Ryff J. V.* Extreme points of some convex subsets of $L^1(0, 1)$. Proc. Amer. Math. Soc. 1967. Vol. 18, pp. 1026–1034.
4. *Ryff J. V.* Majorized functions and measures. Nederl. Akad. Wetensch. Proc. Ser. A. 1968. Vol. 30, pp. 431–437.

I. M. Erusalimskiy (Rostov-on-Don, Russian Federation)
ymerusalimskiy@sfedu.ru , ORCID 0000-0003-4858-6891

ABOUT SPLASHES OF DYNAMIC FLOWS IN NETWORKS

The report will be dedicated to Stefan Samko and his anniversary.

The dynamic flow in the network is considered, i. e. the function defined on the arcs of the network and depending on the discrete time (Z_+). As it is usually, we assumed that:
- at any given time, the flow along any arc does not exceed its throughput capacity.
- at each intermediate vertex of the network, the condition for preserving the flow is met the total flow along the incoming arcs at any time is equal to the total flow along all the outgoing arcs at the next time.

The amount of flow in the network at time $t \in Z_+$ is the sum of flows at time t over all incoming arcs in network drain. The initial flow (at time $t = 0$) is assumed to be equal to zero on all arcs of the network.

The theory of stationary flows in a network, in contrast to the theory of dynamic flows, has been studied for a long time and fully by L. Ford and D. Fulkerson ([1]). In the works [2], [3], [4] we studied a phenomenon called a splash of dynamic flow, when its value exceeds at some time moment the value of the maximum stationary flow.

We considered such dynamic flows in which the flow through the minimum section closest to the drain is stationary and is equal to the throughput of this minimal cut (which is quite natural and corresponds to the ideas of L. Ford, D. Fulkerson), since among such flows there will be those with the maximum value of splashes. A necessary and sufficient condition for the existence of splashes was obtained, consisting in the presence of paths of different lengths leading from this cut to the drain in the part of the network that is located behind the cut (i.e., containing the drain).

An example of a network is constructed in which bursts are possible for such dynamic flows, in which the flow through the minimum cut closest to the drain is stationary, but less than the throughput of this section. For this network, the magnitude of the possible maximum splash is calculated, and as well as the exact boundary that distinguishes in the entire set of dynamic flows, a subset of flows that are guaranteed to have no splashes.

It seems that finding both the first and the second for an arbitrary network is an NP-complete problem.

REFERENCES

1. *Ford Jr. L. R., Fulkerson D. R.* Maximal flow through a network. Canadian J. Math. –1956. –No. 8. – pp. 399–404

2. *Erusalimskiy I. M., Vodolazov N. N.* Non-stationary flow in the network. Bulletin of DSTU. –2009. –v. 9. №3(42). – pp.402–409

3. *Erusalimskiy I. M., Vodolazov N. N.* Maximum network splash and maximum network volume. Izvestiya Vuzov. Severo-Kavkazskii Region. Natural Science.–2010, №6. pp. 9–13

4. *Erusalimskiy I. M.* On the Dynamic Flows in Networks. Universal J. of Communications and Network, 2014, v.2, №6, pp. 101 – 105, DOI: 10.13189

H. G. Feichtinger
(NuHAG, Faculty of Mathematics, University Vienna, Wien, Austria)
hans.georg.feichtinger@univie.ac.at

APPROXIMATION BY TRANSLATES IN INVARIANT BANACH SPACES OF DISTRIBUTIONS AND THE BOUNDED APPROXIMATION PROPERTY

In this talk, we shall present a few recent results concerning the approximation of functions in Banach spaces of distributions (or just measurable functions) by finite linear combinations. The key feature of the approach is the use of integrated group actions (realized as convolution with elements from a Beurling algebra or and pointwise multiplication with respect to some Fourier-Beurling algebra), the use of bounded approximate units, and discretization of such convolution products.

These results have been obtained in joint work with Anupam Gumber (the first paper is about to appear in Proc. AMS soon).

As a by-product (making use of the characterization of compact sets in those Banach spaces, which are double Banach modules) it can be demonstrated that all those (separable) Banach spaces of tempered distributions satisfy the bounded approximation property. Such a statement had been shown already in a widely unknown paper by the author (jointly with W.Braun), published in 1985, using fairly abstract methods.

L. V. Gargyants (Moscow, Russia)
lsteklova@gmail.com

UNBOUNDED SOLUTIONS OF ONE-DIMENSIONAL CONSERVATION LAWS

In a strip $\Pi_T = \{(t,x) \mid t \in (0,T),\ x \in \mathbb{R}\}$, where $0 < T \leq \infty$, we consider the Cauchy problem

$$u_t + (f(u))_x = 0, \quad (t,x) \in \Pi_T, \quad u\big|_{t=0} = u_0(x), \quad x \in \mathbb{R}. \tag{1}$$

In [1] the Cauchy problem (1), with an odd flux function that has a single point of inflexion at zero was studied. A method for constructing sign-alternating discontinuous entropy solutions was proposed.

In the report we consider asymmetrical flux functions of the form

$$f(u) = \begin{cases} C_1|u|^{\alpha-1}u, & u > 0, \\ C_2|u|^{\alpha-1}u, & u < 0, \end{cases} \quad C_1,\, C_2 > 0,\, \alpha > 1. \tag{2}$$

Theorem 1. *The Cauchy problem (1) with flux function (2) and the initial condition* $u_0(x) = e^{-x}$ *has a generalised entropy solution. This solution*

1) is defined in the whole of the half-plane $t > 0$*, and is locally bounded there;*

2) has a countable number of curves of discontinuity which are the graphs of logarithmic functions;

3) is sign-alternating and one-sided periodic with respect to spatial variable.

The statements formulated in theorem 1 are valid when the initial condition has the form

$$u_0(x) = \begin{cases} e^{-x}, & \text{if } x \in (-\infty,\, m), \\ \tilde{u}_0(x), & \text{if } x \in [m,\, +\infty), \end{cases}$$

where $\tilde{u}_0(x)$ is an arbitrary positive nonstrictly increasing piecewise smooth function such that $\tilde{u}_0(m) \geq e^{-m},\ m \in \mathbb{R}$.

This work was supported by the Russian Federation Presidential Council for Grants (grant no. MK-1204.2020.1), and the Ministry of Science and Higher Education of the Russian Federation (project no. 0705-2020-0047).

REFERENCES

1. *Gargyants L. V., Goritsky A. Yu, Panov E. Yu.* Constructing unbounded discontinuous solutions of scalar conservation laws using the Legendre transform. Sb. Math. 2021. Vol. 212, No. 4, pp. 475–489.

A. S. Gasparyan (Pereslavl-Zalesskii, Russia)
armenak.gasparyan@yandex.ru

HYPERCUBIC GENERALIZATIONS OF KANTOROVICH INEQUALITY

In 1948 Kantorovich proved his inequality appeared as following lemm.

Lemm 1. *(L.V.Kantorovich.) There holds the inequality*

$$\sum_k \gamma_k u_k^2 \sum_k \gamma_k^{-1} u_k^2 \leq \frac{1}{4}\left[\sqrt{\frac{M}{m}}+\sqrt{\frac{m}{M}}\right]^2 \left(\sum_k u_k^2\right)^2 \qquad (1)$$

where $0 < m \leq M$, $m \leq \gamma_k \leq M$ *and* $u_k \in R$, $k = 1, 2, \ldots$.

The inequality (1) was generalized in several directions. Greub and Rheinboldt prowed the inequality

$$(x,x)^2 \leq (Ax,x)(A^{-1}x,x) \leq \frac{(M+m)^2}{4Mm}(x,x)^2 \quad \text{for} \quad \forall x \in H, \qquad (2)$$

where A is a bounded positive selfadjoint operator in Hilbert space H, and
$0E \leq mE \leq ME$, E be identity operator, $m, M \in R$.

Strang proved the polarized variant of inequality (2).

$$(Tx,y)(x,T^{-1}y) \leq \frac{(M+m)^2}{4Mm}(x,x)(y,y) \quad \text{for} \quad \forall x,y \in H, \qquad (3)$$

where T is arbitrary linear operator acting in H, and $\|T\| = M$, $\|T^{-1}\| = m^{-1}$.

My talk will be devoted to a family of inequalities on arbitrarily finite sets of sequences, functions or operators. Presented results contain as particular cases the inequalities (1), (2), (3) and some other inequalities related to Kantorovich one.

M. S. Germanchuk, M. G. Kozlova (Simferopol, Russia)
m.german4uk@yandex.ru, art-inf@mail.ru

PSEUDO-BOOLEAN MODELS OF MULTI-AGENT ROUTING

The applied theory of routing problems of the type of many traveling salesman agents ($mTSP$) on complex networks is based on exact solutions of selected classes of problems with polynomial solution algorithms, the use of approximate algorithms and clustering of the original problem, i.e., reduction to problems of smaller dimension and clarifying transformations to return to the original problem. It is important in this process to take into account all available information, knowledge, facts and precedents, both for building a hierarchy of models (extracting models) and for developing practical solution algorithms.

For multi-agent systems of the $mTSP$ type, a pseudo-Boolean conditional optimization model with constraints in the form of disjunctive normal forms (DNF) is used. Such models take into account linear restrictions on the passage of vertices, declarative requirements, the requirements of precedence, the mandatory passage of a selected set of arcs, and other precedent information.

Pseudo-Boolean models with separable objective functions and DNF constraints having a bounded constant length are polynomial solvable. Classes of problems that are reduced or easily reduced to a form with DNF constraints are of interest, since in general such conversions are exponential. The approximate synthesis of a model with DNF constraints from the data has polynomial complexity. It is shown that the number of conjunctions in the extracted DNF does not exceed the number of examples in the original case information. In the works of V. I. Donskoy [1] and M. G. Kozlova [2] proposed algorithms for solving such problems in the presence of incomplete, precedent information, which are used to solve problems of the type $mTSP$.

REFERENCES

1. *Donskoy V. I.* A synthesis of pseudo-Boolean empirical models by precedential information. Vestnik YuUrGU. Ser. Mat. Model. Progr. 2018. Vol. 11. no. 2. pp. 96-107.

2. *Kozlova M. G.* Multi-criteria decision-making models with linear pseudo-Boolean functions and disjunctive constraint. Artificial Intelligence. 2000. no. 2. pp. 67-73.

M. S. Germanchuk, M. G. Kozlova, L. I. Rudenko
(Simferopol, Russia)
m.german4uk@yandex.ru, art-inf@mail.ru, domlir@yandex.ru

AN INTELLECTUALIZED APPROACH IN THE MODELS OF CHOOSING TOURIST ROUTES

Planning routes to tourist attractions leads to various combinations of discrete optimization problems. Based on the data about attractions that are obtained from reliable sources and data on user preferences, it is necessary to make optimal routes for several days, taking into account the opening hours and the desired time of visit. One of the models is the routing model of many traveling salesmen ($mTSP$) in the form of pseudo-Boolean optimization with DNF constraints. In addition to a set of algorithms, the tasks are solved to form a database of attractions with a cartographic reference to the area; to develop an application and a web resource. The server web application must withstand a large amount of calculations (solving a large number of $mTSP$). Indicators are associated with each vertex of the graph (attraction): the number of visiting hours, work schedule, etc. The user sets the dates of the visit, the start and end time, the starting point, etc. The mapping services Google maps and Yandex maps are close to this task, but they allow you to build routes from several points only in the specified order, but they also have the necessary advantages-attractions marked on the map with reliable information, a highly interactive interface for interacting with the map. Some of the features of map services are delivered in the form of a free API, which is used in a web application. At all stages: collecting data about attractions, their ratings; algorithms for building routes; intelligent data processing technologies are used in the development of the interface. The application is based on the actively developing technologies React, TypeScript, Redux, the Yandex maps service. The existing version of the application is a test, but it already provides unique opportunities for planning tourist routes.

REFERENCES

1. *Kozlova M. G.* Synthesis of narrowing enquiries. J. Math. Sc. 2001. Vol. 107. no. 6. pp. 4500-4502. doi:107.10.1023/A:1012593525732.

A. V. Gil (Rostov-on-Don, Russia)
gil@sfedu.ru

An integral operator with a homogeneous kernel in the function space BMO^k

Operators with homogeneous kernels of degree -1 in the space BMO^k, $k \in Z_+$ - functions with a bounded average oscillation of k - that order were considered. Such operators have numerous applications and are well studied in the spaces of summable or smooth functions. In one-dimensional theory, in the spaces $L_p, 1 < p < \infty$, these classes of operators are closely related to convolution operators (using exponential substitutions), as well as to singular integral operators (using the Mellin transform for operators with homogeneous kernels of degree -1). These relationships lead to the correspondence of the **operator** $\Longleftrightarrow$**kernel**$\Longleftrightarrow$**symbol**, which allows us to formulate in terms of the symbol and its index the main properties of operators related to the description of their spectral and Fredholm properties. BMO spaces have long arisen in various issues related to integral operators. It is in their terms, for example, that the image of fractional integrals and Riesz potentials is described in the so-called limit case of Sobolev's theorem, when $\alpha = np$. In terms of belonging to BMO classes of functions are described for which a commutator with a singular integral operator turns out to be compact, etc. Higher-order BMO spaces turn out to be closely related to Besov-type spaces.

The main object considered is an integral operator with a homogeneous kernel of degree -1

$$Kf(x) = \int_a^b h(t)k(x,y)f(y)dy, \qquad x \in (a,b),$$

where $h(t)$ is a continuous function. For the latter operator, the cases are interesting when the singular point $x = 0$ lies on the boundary (that is, the cases of the segment [0,1] or the semi-axis). Sufficient conditions for the boundedness of integral operators K with homogeneous kernels of degree -1 in the spaces of functions with a bounded average oscillation of order k on the semiaxis, on the segment, were obtained, and its spectral properties are studied. It is interesting to note that during the transition to the space BMO^k, conditions, limitations depend on the order of k, and the conditions of Fredholmness for the segment $[0, 1]$ are the same as when $k = 0$, while seemingly for easy operator on the half-axis, they are independent of order k .

Yu. E. Gliklikh, E. S. Zvyagina (Voronezh, Russia)
yeg@math.vsu.ru

STOCHASTIC ANALYSIS AND VISCOUS HYDRODYNAMICS

The talk is devoted to the Lagrangiaan approach to hydrodynamics initiated by well-known works by V.I. Arnold [1] and then by D. Ebin and J. Marsden [2]. In [2], in the language of infinite-dimensional Riemanninan geometry of the Sobolev diffeomorphisms groups on compact manifolds there was given the description of ideal incompressible fluids. In particular, it was shown that the flow of ideal incompressible fluid with zero external force is described by the equation of geodesic of weak Riemannian metric of the group of volume preserving diffeomorphisms.

We show that the flows of viscous incompressible fluids are described by stochastic analogues of the Ebin and Marsden equations, in which the covariant derivative is replaced by the second order backward mean derivatives. In spite of the fact that the construction is based on the Stochastic Analysis, the results are obtained for deterministic (not random) fluids. Unlike Ebin and Marsden, we consider the hydrodynamics only on the flat n-dimensional torus. Investigation of fluid motion on the torus is a well-known problem in the hydrodynamics.

We construct an analogue of the Wiener process on the group of diffeomorphisms based on the Wiener process on the torus and describe the stochastic differential equations with mean derivatives mentioned above, in terms of this process. Then we show that the expectation of solution of such equation, after transition to the Euler approach, satisfies the Navier-Stokes equation. After that we show that the solution of the above-mentioned equation on the group of diffeomorphisms can be constructed form the flow of ideal incompressible flow on the torus by changing the time direction and application of the left shift by the above mentioned Wiener process.

The machinery of mean derivatives can be found in [3].

REFERENCES

1. *Arnold V.* Sur la géometrie différentielle des groupes de Lie de dimension inÏnie et ses applications a l'hydrodynamique des fuides parfaits. Ann. Inst. Fourier. 1966. Vol. 16, pp. 319-361.

2. *Ebin D. G., Marsden J.*: Groups of diffeomorphisms and the motion of an incompressible fluid. Annals of Math. 1970. Vol. 92, No. 1, pp. 102–163.

3. *Gliklikh Yu. E.* Global and stochastic analysis with applications to mathematical physics. Springer-Verlag London. 2011.

V. P. Golubyatnikov, N. E. Kirillova (Novosibirsk, Russia)
golubyatn@yandex.ru

A MODEL OF ONE CIRCADIAN OSCILLATOR

We consider 6D nonlinear dynamical system

$$\dot{x}_1 = k_1(\Gamma_1(x_2)\cdot\gamma_1(x_3) - x_1);\;\; \dot{x}_j = k_j(\Gamma_j(x_5)\cdot L_j(x_1) - x_j);\; j = 2,3,4;$$
$$\dot{x}_5 = k_5(\Gamma_5(x_6) - x_5);\;\; \dot{x}_6 = k_6(C\cdot L_6(x_4) - x_6); \tag{1}$$

as a model of circadian oscillator proposed in [1,2].

Here $C > 0$ is a constant, the variables x_m describe concentrations of components, coefficients $k_m > 0$ describe their natural degradations; smooth functions γ_1, $\Gamma_m > 0$ increase monotonically (positive feedbacks), and smooth positive functions L_m are monotonically decreasing, they correspond to negative feedbacks in this gene network. Analytic forms of all these functions are not specified in our studies at all.

We find sufficient conditions of uniqueness of an equilibrium point S_0 of the system (1): *the functions* $\Gamma_j(x_5)$ *are proportional, the functions* $L_j(x_1)$ *are proportional,* $j = 2,3,4$. Let all k_m be equal.

Theorem. *If* $-CL_6'\Gamma_5L_4\Gamma_4' > 8 - 2(\Gamma_1\gamma_1'L_3'\Gamma_3 + \Gamma_1'\gamma_1L_2'\Gamma_2)$, *and all these conditions above are satisfied, then the equilibrium point* S_0 *is unstable, and the system* (1) *has a cycle in the positive octant* $\mathbb{R}^6_+$.

All the functions and their derivatives here are calculated at the point S_0. Note that $L_m' < 0$ for all m.

In other words, the cycle does exist if the rates of the synthesis of the components described by the 4-th, 5-th and 6-th equations of the system (1) exceed those in the first three equations. These two subsystems correspond to two distinct parts of this gene network. 3D-simplifications of this model do not have cycles, see [3].

The authors are indebted to O.A.Podkolodnaya, and N.L.Podkolodnyy for helpful discussions and biological interpretations. The work is supported by RFBR, grant 20-31-90011.

REFERENCES

1. *Podkolodnaya O. A., Tverdokhleb N. N., Podkolodnyy N. L.* Computational modeling of the cell autonomous mammalian circadian oscillator, BMC Systems Biology. 2017. Vol. 11, pp. 27–42.

2. *Almeida S., Chaves M., Delaunay F.* Transcription-based circadian mechanism controls the duration of molecular clock states in response to signaling inputs. Journal of Theoretical Biology. 2020. Vol. 484, pp. 110015.

3. *Golubyatnikov V. P., Kirillova N. E.* Phase portraits of two gene networks models (in Russian). Mathem. Notes of NEFU. 2021. Vol. 28, No. 1, pp. 3–11.

O. A. Ivanova (Rostov on Don, Russia)
neo_ivolga@mail.ru

THE COMMUTANT OF THE GENERALIZED BACKWARD SHIFT OPERATOR AND THE DUHAMEL PRODUCT

Let Q be a convex domain in $\mathbb{C}$, containing the point 0; $H(Q)$ be the Fréchet space of all holomorphic functions on Q; E_Q be a countable inductive limit of weighted Banach spaces which with the help of Laplace transform is topologically isomorphic to the strong dual of $H(Q)$. A function $g_0 \in E_Q$ such that $g_0(0) = 1$ defines the generalized backward shift operator

$$D_{0,g_0}(f)(t) = \frac{f(t) - g_0(t)f(0)}{t},$$

which is continuous and linear in E_Q. If $g_0 \equiv 1$, then D_{0,g_0} is the usual backward shift operator. It is assumed that g_0 has a finite number of zeros or has no zeros. We investigate continuous linear operators in E_Q, which commute with D_{0,g_0} in E_Q. Such operators have been described in [1]. Necessary and sufficient conditions are proved that an operator of the mentioned commutant is a topological isomorphism of E. The problem of the factorization of nonzero operators of this commutant is investigated. In the case when the function g_0, defining the generalized backward shift operator, has zeros in Q, they are divided into two classes: the first one consists of isomorphisms and surjective operators with a finite-dimensional kernel, and the second one contains finite-dimensional operators. In the proofs we use essentially a characterization [2] of proper closed D_{0,g_0}-invariant subspaces of E_Q. With the help of obtained results we study the generalized Duhamel product in $H(Q)$. If $g_0 \equiv 1$, then it is the Duhamel product, which in $H(Q)$ has been introduced and studied by N. Wigley [3]. The above results are proved in [4].

REFERENCES

1. *Ivanova O. A., Melikhov S. N.* On operators which commute with the Pommiez type operator in weighted spaces of entire functions. St. Petersburg Math. J. 2017. Vol. 28, No. 2, pp. 209–224.

2. *Ivanova O. A., Melikhov S. N.* On invariant subspaces of the Pommiez operator in the spaces of entire functions of exponential type. J. of Math. Sci. 2019. Vol. 241, No. 6, pp. 760–769.

3. *Wigley N.* The Duhamel product of analytic functions. Duke Math. J. 1974. Vol. 41, pp. 211-217.

4. *Ivanova O. A., Melikhov S. N.* On the commutant of the generalized backward shift operator in weighted spaces of entire functions. Trends in Mathematics: Operator Theory and Differential Equations. Editors: A. G. Kusraev, Z. D. Totieva. Birkhäuser. 2021, pp. 37–48.

M. A. Karapetyants (Moscow, Russia)
karapetytantsmk@gmail.com

DYADIC ANALOGUE OF THE PAUL ERDOS PROBLEM

We are exploring the dyadic analogue of one of the Paul Erdos problem, namely, the existence of a probability density of a random variable (which is a power series), extended to a dyadic half-line. We consider the power series with coefficients being either zeroes or ones at the fixed point x of the $(0, 1)$ interval. The question is whether there is a density from $\mathbb{L}_1$? In classic case it is still an open problem for x greater than one half (P. Erdos proved the non-existence of the density for lambdas equal to $\frac{1}{p}$, where p is the Pisot number). Moreover, we study the so called "dual problem". The same random variable, but the point x is fixed now ($x = \frac{1}{2}$) and the coefficients are integer and belong to $[0; N]$ segment for some natural N. Here we answer the same question and provide criteria of the existence of a density in terms of the solution of the refinement equation as well as in terms of the coefficients of a random variable.

REFERENCES

1. *P. Erdos* On the smoothness properties of Bemuolli convolutions, Am. J. Math., 62, 180-186.
2. *P. Erdos* On a family of symmetric Bernuolli convolutions, Am. J. Math., 61, 974-975.
3. *V. Protasov* Refinement equations with nonnegative coefficients, The Journal of Fourier Analysis and Applications 6, 55–78 (2000).
4. *Golubov B. I., Efimov A. V., Skvortsov V. A.* Series and Walsh transform. Theory and applications, Science, 1987.
5. *Protasov V. Yu., Farkov Yu. A.* Dyadic wavelets and refinable functions on a half-line, Sbornic: Mathematics. 2006. Vol. 197, No. 10, pp. 129-160.

K. S. Kazarian (Madrid, Spain)
kazaros.kazarian@uam.es

HARDY SPACES WITH ADMISSIBLE WEIGHTS

We say that a weight function w defined on $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$ is an admissible weight function if $\ln w$ is integrable. For a given admissible weight function w and $1 \leq p < \infty$ a Hardy space $H^p(w)$ is the closed linear span in $L^p(\mathbb{T}, w)$ of the of the system $\{e^{int}\}_{n=0}^{\infty}$. From G. Szegö's theorem (see [1],[2]) it folows that $H^p(w)$ is a strict subspace of $L^p(\mathbb{T}, w)$. We characterize the dual spaces of $H^p(w), 1 \leq p < \infty$. We prove that $\{e^{int}\}_{n=0}^{\infty}$ is an $M-$basis in $H^p(w)$. Weight functions for which the system $\{e^{int}\}_{n=0}^{\infty}$ is an $A-$ summation basis in $H^p(w), 1 \leq p < \infty$ are characterized.

A function f holomorphic in the open unit disk $\mathbb{D}$ belongs to $\mathcal{H}^p(w), 1 \leq p < \infty$ if

$$\|f\|^p_{\mathcal{H}^p(w)} = \sup_{0 \leq r < 1} \int_{\mathbb{T}} |f(re^{it})|^p w(t) dt < \infty.$$

Related results for weighted Hardy spaces $\mathcal{H}^p(w)$, $1 \leq p < \infty$ are obtained.

REFERENCES

1. *Grenander U. and Szegö G.* Toeplitz forms and their applications. University of California Press. 1958.
2. *Hoffman K.* Banach spaces of analytic functions. Englewood Cliffs: Prentice-Hall. 1962.

V. Kiryakova (Sofia, Bulgaria)
virginia@diogenes.bg

THE MULTI-INDEX MITTAG-LEFFLER FUNCTIONS AS SPECIAL FUNCTIONS OF FRACTIONAL CALCULUS

The developments in theoretical and applied science have always required knowledge of the properties of the "mathematical functions" (in terms of Bateman), from elementary trigonometric functions to the variety of Special Functions (SF), appearing whenever natural and social phenomena are studied, engineering problems are formulated, and numerical simulations are performed. The well known "Classical SF" (SF of Mathematical Physics, Named SF) are related to differential and integral equations of *integer order*, mainly of 2nd order, but also of higher ones.

With the recognition that the fractional order models can describe better the fractal nature or our world, the solutions of the *fractional order differential and integral equations and systems* also gained their important place. Among them, the *Mittag-Leffler (M-L) function* $E_{\alpha,\beta}(z)$ as long time ignored in handbooks on SF, exited from its isolated life as Cinderella of the SF and was honored as the Queen function of FC (Gorenflo and Mainardi). But there is a galaxy of what we call now as "*SF of Fractional Calculus*" (*SF of FC*), the most general ones being the Fox H-function and the generalized Wright function ${}_p\Psi_q(z)$, providing a lot of examples of solutions to fractional order models. Among them, important place have the extensions of the M-L function, called M-L type functions or *multi-index M-L functions*, where additional and even vector-type parameters are involved so to encompass a bigger variety of related fractional and integer order/multi-order operators of Calculus.

In this talk we provide some short overview on the definitions, basic properties and unexpectedly long list of examples of the multi-index M-L functions $E^{(m)}_{(\alpha_i),(\beta_i)}(z)$, studied by the author in the recent two decades.

REFERENCES

1. *Kiryakova V.* The multi-index Mittag-Leffler functions as important class of special functions of fractional calculus. Computers and Math. with Appl. 2010. Vol. 59, pp. 1885–1895; doi:10.1016/j.camwa.2009.08.025.

2. *Kiryakova V.* Unified approach to fractional calculus images of special functions-A survey. Mathematics (MDPI). 2020. Vol. 8, Art. 2260, pp. 1–35; doi:10.3390/math8122260.

3. *Kiryakova V.* A guide to special functions in fractional calculus. Mathematics (MDPI). 2021, Vol. 9, Art. 106; pp. 1–35; doi:10.3390/math9010106.

Yu. V. Korablina (Rostov-on-Don, Vladikavkaz, Russia)
anaconda210150@mail.ru

ON CONDITIONS OF BOUNDEDNESS OF LINEAR OPERATORS ON WEIGHTED QUASI-BANACH SPACES

We consider a problem of boundedness of classical operators acting on weighted quasi-Banach spaces of holomorphic functions $H_v(G)$. This space is defined as follows:

$$H_v(G) = \{f \in H(G), \|f\|_v = \sup_{z\in G} \frac{|f(z)|}{v(z)} < \infty\}.$$

Here, G is a domain in the complex plane $\mathbb{C}$, $H(G)$ is the space of all holomorphic functions in G, and v is a weight on G.

Here and bellow $X \hookrightarrow H(G)$ is quasi-Banach space, endowed with the quasi-norm $\|\cdot\|$. X^* is a dual space with X of linear intrinsic functionals on X, endowed with the norm $\|\cdot\|^*$. δ_z is δ-function for a fixed point $z \in G$.

Theorem 1. Let v be an arbitrary weight on G. Linear operator $T : X \mapsto H_v(G)$ is bounded if and only if

$$\text{a) } \delta_z(T) \in X^* \text{ for all } z \in G; \quad \text{b) } \sup_{z\in G} \frac{\|\delta_z(T)\|^*}{v(z)} < \infty.$$

This result allowes to establish some criteria of the boundedness of weighted composition operator and Volterra operator on abstract quasi-Banach space in terms of δ-functions. As a consequence we obtain criteria of the boundedness of the above mentioned operators on generalized Bergman, Bloch and Fock spaces.

R E F E R E N C E S

1. *Zorboska N.* Intrinsic operators from holomorphic function spaces to growth spaces. Integr. Equ. Oper. Theory.—2017.—V. 87, № 4.—P. 581–600.

2. *Abanin A.V., Pham Trong Tien.* Differentiation and integration operators on weighted spaces of holomorphic functions. Math. Nachr.—2017.—V. 290, № 8–9.—P. 1144–1162.

A. A. Kornuta, V. A. Lukianenko, (Simferopol, Russia)
korn_57@mail.ru
art-inf@yandex.ru

SCENARIOS OF THE BEHAVIOR OF SOLUTIONS OF A NONLINEAR FUNCTIONAL-DIFFERENTIAL EQUATION OF PARABOLIC TYPE WITH TRANSFORMATION OF ARGUMENTS

We consider an initial-boundary value problem for a nonlinear functional-differential equation of parabolic type, which demonstrates the behavior of the phase modulation $u(x,t)$ of a light wave passing through a thin layer of a nonlinear Kerr-type medium within the aperture $S \subset \mathbb{R}^2$ in optical system with feedback loop:

$$\frac{\partial u(x,t)}{\partial t} + u(x,t) = \mu \triangle u(x,t) + K(1 + \gamma \cos Qu(x,t)),\ x \in S,\ t \geq 0,$$

here $\triangle$ – Laplace operator, $\mu > 0$ – diffusion coefficient of particles of a nonlinear medium, $Qu(x,t) = u(q(x),t)$ – operator of rotation by angle h ($Q^m = I$ [1]), $K > 0$ – coefficient, nonlinearity, which is proportional to the intensity of the incoming stream, γ ($0 < \gamma < 1$) – coefficient of visibility (contrast) of an interference pattern [2].

In [3] particular cases of solutions of this problem for a ring are described in detail. Using the method of central manifolds and the Galerkin method, scenarios of the emergence of spatially inhomogeneous structures and structures of the "traveling wave" type bifurcating spatially homogeneous solutions of this equation that change the character of stability, are described for circular regions (circle, ring) [4].

REFERENCES

1. *Karapetyants N. K., Samko S. G.* Equations with invulative operators and their applications. Rostov: Rostov University Press, 1988.

2. *Razgulin A. V.* Nonlinear models of optical synergetics. M: MGU, MAKS Press, 2008. 203 p.

3. *Kornuta A. A., Lukianenko V. A.* Functional differential equations of parabolic type with an involution operator. Dinam. Sist. 2019. 9 (37), P. 390–409.

4. *Kornuta A. A., Lukianenko V. A.* Stable Structures of Nonlinear Parabolic Equations with Transformation of Spatial Variables. Lobachevskii J Math. 2021. Vol. 42, pp. 911–930.

A. A. Kovalevsky (Krasovskii Institute of Mathematics and Mechanics and Ural Federal University, Yekaterinburg, Russia)
alexkvl71@mail.ru

ON THE CONVERGENCE OF SOLUTIONS OF VARIATIONAL PROBLEMS WITH VARIABLE REGULAR BILATERAL CONSTRAINTS IN VARIABLE DOMAINS

In this talk, we consider a sequence of convex integral functionals $F_s\colon W^{1,p}(\Omega_s) \to \mathbb{R}$ and a sequence of weakly lower semicontinuous and generally non-integral functionals $G_s\colon W^{1,p}(\Omega_s) \to \mathbb{R}$, where $\{\Omega_s\}$ is a sequence of domains in $\mathbb{R}^n$ contained in a bounded domain $\Omega \subset \mathbb{R}^n$ $(n \geqslant 2)$ and $p > 1$. Along with this, we consider the sequence of sets

$$V_s = \{v \in W^{1,p}(\Omega_s)\colon \varphi_s \leqslant v \leqslant \psi_s \text{ a.e. in } \Omega_s\},$$

where φ_s and ψ_s are functions in $W^{1,p}(\Omega_s)$ such that $\varphi_s \leqslant \psi_s$ a.e. in Ω_s. We describe conditions for the convergence of minimizers and minimum values of the functionals $F_s + G_s$ on the sets V_s. These conditions include the strong connectedness of the sequence of spaces $W^{1,p}(\Omega_s)$ with the space $W^{1,p}(\Omega)$, the Γ-convergence of the sequence $\{F_s\}$ to a functional $F : W^{1,p}(\Omega) \to \mathbb{R}$, and a certain convergence of the sequence $\{G_s\}$ to a functional $G : W^{1,p}(\Omega) \to \mathbb{R}$. As for the constraints φ_s and ψ_s, our main condition is that $\operatorname{meas}\{\psi_s - \varphi_s < \alpha\} \to 0$ for a positive measurable function $\alpha\colon \Omega \to \mathbb{R}$. We state a result which shows that, under the specified conditions and some additional assumptions on the domain Ω, the integrands of the functionals F_s, and the constraints φ_s and ψ_s, subsequences of minimizers and minimum values of the functionals $F_s + G_s$ on the sets V_s converge to a minimizer and the minimum value of the functional $F + G$ on a set of the form

$$V = \{v \in W^{1,p}(\Omega)\colon \varphi \leqslant v \leqslant \psi \text{ a.e. in } \Omega\},$$

where $\varphi, \psi \in W^{1,p}(\Omega)$. A detailed description of this and other related results is given in [1].

REFERENCES

1. *Kovalevsky A. A.* Variational problems with variable regular bilateral constraints in variable domains. Rev. Mat. Complut. 2019. Vol. 32, No. 2, pp. 327–351.

Nadezhda P. Krasii (Russia, Rostov-on-Don, DSTU)
krasnad@yandex.ru
Pedro P. Mota (Portugal, New University of Lisbon)
pjpm@fct.unl.pt
Manuel L. Esquível (Portugal, University of Lisbon)
mle@fct.unl.pt

STOCK PRICE: DOUBLE DIFFUSION MODELS

Regime switching diffusion processes with one or two thresholds and regime switching occurring by a change in the diffusion drift and/or volatility functions parameters of a stochastic differential equation, whose solution defines a continuous time diffusion process, were defined in previous works; the change in regime occurring whenever the trajectory of the process crosses a threshold, possibly with some delay. In this paper we generalise the previous results by allowing the underlying diffusion process to change from one family of diffusions in one regime to an entirely different one in the other regime; these families of diffusions are characterised by specific functional forms for drift and volatility coefficients depending on parameters. We propose an estimation procedure for all the parameters, namely the thresholds, the delay and, for both regimes, diffusion's parameters and we apply the introduced estimation procedure to both simulated and real data.

FUNDING: For the first author, this work was done under partial financial support of RFBR (Grant n. 19-01-00451).

R E F E R E N C E S

1. *Mota P. P., Krasii N. P., Esquível M. L.* Some Double Diffusion Models For Stock Prices. Global and Stochastic Analysis 2021. Vol. 8, No. 2, pp. 53–66.

2. *Esquível M. L., Krasii N. P., Mota P. P.* Auto and externally induced regime switching diffusions. Communications on Stochastic Analysis, 2020. Vol. 14, No. 1-2, pp. 31–53.

3. *Mota P. P., Esquível M. L.* Model selection for stock prices data. Journal of Applied Statistics 2016. Vol. 43, No. 16, pp 2977–2987.

4. *Mota P. P., Esquível M. L.* On a continuous time stock price model with regime switching, delay, and threshold. Quantitative Finance 2014. Vol. 14, No. 8, pp. 1479–1488.

5. *Esquível M. L., Mota P. P.* On some auto-induced regime switching double-threshold glued diffusions. Journal of Statistical Theory and Practice, 2014. Vol. 8, No. 4, pp. 760–771.

L. G. Kurakin (Moscow, Vladikavkaz, Russia),
A. V. Kurdoglyan (Vladikavkaz, Russia)
aik_kurdoglyan@mail.ru

BIFURCATION ANALYSIS IN A NEIGHBORHOOD OF A COSYMMETRIC EQUILIBRIUM USING THE LYAPUNOV-SCHMIDT METHOD

A dynamical system with a cosymmetry is considered. V. I. Yudovich showed that a noncosymmetric equilibrium of such a system under the conditions of the general position is a member of a one-parameter family [1, 2]. In this paper, it is assumed that the equilibrium is cosymmetric, and the linearization matrix of the cosymmetry is nondegenerate. It is shown that, in the case of an odd-dimensional dynamical system, the equilibrium is also nonisolated and belongs to a one-parameter family of equilibria. In the even-dimensional case, the cosymmetric equilibrium is, generally speaking, isolated. The Lyapunov – Schmidt method is used to study bifurcations in the neighborhood of the cosymmetric equilibrium when the linearization matrix has a double kernel. The dynamical system and its cosymmetry depend on a real parameter. We describe scenarios of branching for families of noncosymmetric equilibria. The results are published in [3].

The work of the first author was carried out in the frame of the Program No. 0147-2019-0001 (State Registration No. AAAA-A18-118022090056-0).

REFERENCES

1. *Yudovich V. I.* Cosymmetry, degeneration of solutions of operator equations, and onset of a filtration convection. Math. Notes. 1991. Vol. 49, No. 5–6, pp. 540–545.

2. *Yudovich V. I.* Secondary cycle of equilibria in a system with cosymmetry, its creation by bifurcation and impossibility of symmetric treatment of it. Chaos. 1995. Vol. 5, No. 2, pp. 402–411.

3. *Kurakin L. G., Kurdoglyan A. V.* On the Isolation/Nonisolation of a Cosymmetric Equilibrium and Bifurcations in its Neighborhood. Regular and Chaotic Dynamics, 2021, Vol. 26, No. 3, pp. 258–270.

M. A. Kuznetsova (Saratov, Russia)
kuznetsovama@info.sgu.ru

INVERSE SPECTRAL PROBLEMS FOR STURM-LIOUVILLE OPERATORS ON TIME SCALES

Consider the following Sturm-Liouville equation with Δ-derivatives:

$$-y^{\Delta\Delta}(x) + q(x)y(\sigma(x)) = \lambda y(\sigma(x)), \quad x \in T^{0^2},$$

where T is a time scale and $q(x) \in C(T^{0^2})$ is a real-valued potential. The necessary definitions from the theory of Δ-differentiability can be found in [1]. In [1], the following spectral characteristics were introduced and studied: the eigenvalues $\{\lambda_{n0}\}_{n\geq 1}$ and $\{\lambda_{n1}\}_{n\geq 1}$ of two boundary value problems with a common boundary condition, the Weyl function $M(\lambda)$, and the weight numbers $\{\alpha_n\}_{n\geq 1}$.

Let T consist of N segments and M points:

$$T = \bigcup_{l=1}^{N+M} [a_l, b_l], \quad a_{l-1} \leq b_{l-1} < a_l \leq b_l,\ l = \overline{2, N+M},$$

where $a_l < b_l$ if and only if $l \in \{l_k\}_{k=1}^N$. This class of time scales is the most general one on which inverse problems were studied up to now.

Theorem 1. *Suppose* $q \in W_1^1[a_{l_k}, b_{l_k}], \quad k = \overline{1, N}$. *Then,* q *is uniquely determined by any of the following spectral data sets:*

1. $M(\lambda)$;
2. $\{\lambda_{nj}\}_{n\geq 1}$, $j = 0, 1$;
3. $\{\lambda_{n1}\}_{n\geq 1}$ *and* $\{\alpha_n\}_{n\geq 1}$.

By the method of spectral mappings, taking into account spectral characteristics properties, the author also obtained an algorithm for recovering the potential, see [2].

This work was supported by Grant 20-31-70005 of the Russian Foundation for Basic Research.

REFERENCES

1. *Kuznetsova M. A.* A uniqueness theorem on inverse spectral problems for the Sturm-Liouville differential operators on time scales. Results in Mathematics. 2020. Vol. 75, No. 44, pp. 1–23.

2. *Kuznetsova M. A.* On recovering the Sturm-Liouville differential operators on time scales. Mathematical Notes. 2021. Vol. 109, No. 1, pp. 74–88.

K. S. Lapin (Saransk, Russia)
klapin@mail.ru

GUIDING FUNCTIONS AND THE EXISTENCE OF POISSON BOUNDED SOLUTIONS [1]

We consider an arbitrary system of differential equations of n variables

$$\frac{dx}{dt} = F(t,x), \quad F(t,x) = (F_1(t,x), \dots, F_n(t,x))^T, \qquad (1)$$

which right-hand side is given and continuous in $\mathbb{R}^+ \times \mathbb{R}^n$. It is assumed, that $F(t,x)$ satisfied Lipschitz condition on x. Each increasing numerical sequence $\tau = (\tau_i)$, where $\tau_i \geqslant 0$, $i \geqslant 1$, $\lim\limits_{i\to\infty} \tau_i = +\infty$ we will call $\mathcal{P}$-sequence. For each $\mathcal{P}$-sequence $\tau = \{\tau_i\}_{i\geqslant 1}$ we will denote by $M(\tau)$ the set $\bigcup\limits_{i=1}^{\infty} [\tau_{2i-1}; \tau_{2i}]$. Recall, that solution $x(t, t_0, x_0)$ of system (1) is said to be Poisson bounded [1], if for this solution there exist $\mathcal{P}$-sequence $\tau = \{\tau_i\}_{i\geqslant 1}$, where $t_0 \in M(\tau)$ and there exist number $\beta > 0$ such that the condition $\|x(t,t_0,x_0)\| \leqslant \beta$ for all $t \in R^+(t_0) \bigcap M(\tau)$ holds, where $\mathbb{R}^+(t_0) = \{t \in \mathbb{R} \mid t \geqslant t_0\}$. Continuously differentiable function $G : \mathbb{R}^n \to \mathbb{R}$ is called [2] guiding function or, more precisely, r_0-guiding function for the system (1), if $(\mathrm{grad}\, G(x), F(t,x)) > 0$ for all $t \geqslant 0$ and $\|x\| \geqslant r_0$. Recall, that the r_0-guiding function G for the system (1) is called unbounded if the condition $G(x) \to +\infty$ is satisfied for $\|x\| \to +\infty$.

Theorem 1. *Let for the system* (1) *there exist $\mathcal{P}$-sequence $\tau = (\tau_i)$, non-increasing function $b : \mathbb{R}^+ \to \mathbb{R}^+$, $b(r) \to +\infty$ for $r \to +\infty$, and the vector Lyapunov function $V(t,x)$ with the comparison system $\dot{\xi} = f(t,\xi)$, where $f(t,\xi)$ satisfies the local Lipschitz condition by $\xi \in \mathbb{R}^k$, that for any $(t,x) \in M(\tau) \times \mathbb{R}^n$ the inequality $b(\|x\|) \leqslant \sum\limits_{i=1}^{k} V_i(t,x)$ holds. Moreover, let there exist numbers $r_1 > r_0$ and unbounded r_0-guiding function G for system $\dot{\varrho} = g(t,\varrho)$, where $g(t,\varrho) = f(t, \varrho + \overline{r}_1)$ and $\overline{r}_1 = (r_1, \dots, r_1) \in \mathbb{R}^k$, and let the number r_1 satisfies the following conditions:*

1) $G(\varrho) \geqslant M_0$ for all $\varrho \in \mathbb{R}^k$, $\|\varrho\| = r_1$, where $M_0 = \max_{\|\varrho\|\leqslant r_0} G(\varrho)$.

2) $B^k_{\overline{r}_1}(r_1) = \{\xi \in \mathbb{R}^k \mid \|\,\xi - \overline{r}_1\| \leqslant r_1\} \subset \mathrm{Im}\,(V : \{0\} \times \mathbb{R}^n \to \mathbb{R}^k)$.

Then the system (1) *has at least one Poisson bounded solution.*

REFERENCES

1. *Lapin K. S.* Uniform boundedness in the sense of Poisson of solutions of systems of differential equations and Lyapunov vector functions, Differential Equations, 2018, Vol. 54, No. 1, pp. 38–48.

2. *Krasnoselskii M. A.* Shift operator along the trajectories of differential equations. M.:Nauka, 1966.

[1]This work was supported by the grant of the President of Russia MK-211.2020.1

V. L. Litvinov (Lomonosov Moscow State University, Moscow)
(Samara State Technical University, Samara)
vladlitvinov@rambler.ru

MATHEMATICAL MODEL OF LONGITUDINAL-TRANSVERSE VIBRATIONS OF A BEAM WITH A MOVING BOUNDARY

Until now, the problems of longitudinal - transverse vibrations of objects with moving boundaries were solved mainly in a linear setting, the energy exchange through the moving boundary and the interaction between longitudinal and transverse vibrations were not taken into account [1–5, 7–10]. In rare cases, the action of the forces of resistance of the external environment was taken into account [6]. Real technical objects are much more complicated, for example, with an increase in the intensity of oscillations, the geometric nonlinearities of the object have a great influence on the oscillatory process. In connection with the intensive development of numerical methods, it became possible to more accurately describe complex mathematical models of longitudinal-transverse oscillations of objects with moving boundaries, taking into account a large number of factors influencing the oscillatory process. The talk presents a new nonlinear mathematical model of longitudinal-transverse vibrations of a beam with a moving boundary, which takes into account geometric nonlinearity, viscoelasticity, energy exchange across the boundary. The boundary conditions are obtained in the case of interaction between the parts of the object to the left and to the right of the moving boundary. The resulting model is linearized. In this case, the principle of homogeneity is observed: in the particular case of small fluctuations, the obtained linear models coincided with the classical ones, which indicates the correctness of the results obtained. The obtained mathematical model allows one to describe high-intensity vibrations of a beam with a moving boundary.

REFERENCES

1. *Savin G. N., Goroshko O. A.* Dynamics of a variable length thread // Nauk. dumka, Kiev, 1962, 332 p.

2. *Samarin Yu. P.* On a nonlinear problem for the wave equation in one-dimensional space // Applied Mathematics and Mechanics. - 1964. - T. 26, V. 3. - P. 77–80.

3. *Vesnitsky A. I.* Waves in systems with moving boundaries and loads // Fizmatlit, Moscow, 2001, 320 p.

4. *Lezhneva A. A.* Bending vibrations of a beam of variable length // Izv. Academy of Sciences of the USSR. Rigid Body Mechanics. - 1970. - No. 1. - P. 159-161.

5. *Litvinov V. L* Solution of boundary value problems with moving boundaries using an approximate method for constructing solutions of integro-differential equations // Tr. Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences. 2020.Vol. 26, No. 2. P. 188-199.

6. *Anisimov V. N., Litvinov V. L.* Mathematical models of longitudinal-transverse vibrations of objects with moving boundaries // Vestn. Himself. tech. un-t. Ser. Phys and mat. science, 2015. Vol. 19, No. 2. P. 382-397.

7. *Anisimov V. N., Litvinov V. L.* Mathematical modeling and study of the resonance properties of mechanical objects with a changing boundary: monograph - Samara: Samar. state tech. un – t, 2020. - 100 p.

8. *Litvinov V. L., Anisimov V. N.* Application of the Kantorovich - Galerkin method for solving boundary value problems with conditions on moving boundaries // Bulletin of the Russian Academy of Sciences. Rigid Body Mechanics. 2018. No. 2. P. 70–77.

9. *Litvinov V. L., Anisimov V. N.* Transverse vibrations of a rope moving in a longitudinal direction // Bulletin of the Samara Scientific Center of the Russian Academy of Sciences. 2017. T. 19. No. 4. – P.161–165.

10. *Litvinov V. L., Anisimov V. N.* Mathematical modeling and research of oscillations of one–dimensional mechanical systems with moving boundaries: monograph / V. L. Litvinov, V. N. Anisimov – Samara: Samar. state tech. un–t, 2017 .– 149 p.

V. A. Lukianenko (Simferopol, Russian Federation)
art-inf@yandex.ru

RESOLUTION OF EQUATIONS WITH URYSOHN OSCILLATING OPERATORS

Urysohn-type equations of the first kind arise in various applied problems. Approximate solutions are constructed using asymptotic methods, regularization methods. The model equation is considered

$$A(f,\tau) \equiv \int_{\mathbb{R}} f(\xi - s) n(t - \tau(s)) ds = u(t,\xi), \quad t,\xi \in \mathbb{R} \tag{1}$$

with the kernel $n(t)$ in the form of a delta-type function. Both the function f and the function τ can be searched for. Indirectly, the nature of the nonlinearities $\tau(s)$ in the form of the right part can be determined as a result of solving the spectral problem $Af = \lambda f$ for the linear operator A in f. The application of the Fourier transform on the variable t leads to (1) to the Urysohn equation with an oscillating operator.

$$A_w(t,\tau) \equiv \int_{\mathbb{R}} f(\xi - s) e^{iw\tau(s)} ds = N^{-1}(w)\mathcal{U}(w,\xi) \equiv \mathcal{V}(w,\xi), \tag{2}$$

where $N(w) = (\mathcal{F}n)(w)$, $\mathcal{U}(w,\xi) = (\mathcal{F}u)(w,\xi)$. The equations (1), (2) on $\mathbb{R}$ and on the segment are considered. Iterative, regularizing algorithms are used. For the equation (2) on the segment, the Proni method is used, reducing (2) to the nonlinear equation $\sum\limits_{k=1}^{p} h_k z_k^{n-1} = \mathcal{V}_n$, $n = \overline{1,p}$ and then to the difference equation and the polynomial factorization problem. The characteristic polynomial allows us to classify the contribution of nonlinearity $\tau(s)$. It is shown that for $w \gg 1$ it is necessary to apply asymptotic methods.

REFERENCES

1. *Belozub V., Kozlova M., Lukianenko V.* Approximated solution algorithms for Urysohn-type equations. J. Phys.: Conf. Ser. 2021. Vol. 1902, pp. 012051. doi:10.1088/1722-6596/1902/1/0122051.

A. R. Mirotin (Gomel, Belarus)
amirotin@yandex.ru

CAN ANY NONTRIVIAL HAUSDORFF OPERATOR BE A RIESZ OPERATOR?

We accept the following generalized definition of a Hausdorff operator.

Definition (cf. [1], [2]). Let (Ω, μ) be a σ-compact topological space endowed with a positive regular Borel measure μ, let Φ be a locally integrable function on Ω, and let $(A(u))_{u\in\Omega}$ be a μ-measurable family of $n \times n$-matrices, non-singular for μ-almost every u, with $\Phi(u) \neq 0$. We define the *Hausdorff operator* with the kernel Φ by (recall that $x \in \mathbb{R}^n$ is a column vector)

$$(\mathcal{H}_{\Phi,A}f)(x) = \int_{\Omega} \Phi(u)f(A(u)x)d\mu(u).$$

The problem of compactness of Hausdorff operators was posed by Liflyand [1]. There is a conjecture that nontrivial Hausdorff operator on $L^p(\mathbb{R}^n)$ is non-compact. For the case $p = 2$ and for commuting $A(u)$ this hypothesis was confirmed in [3]. Moreover, we conjecture that a nontrivial Hausdorff operator on $L^p(\mathbb{R}^n)$ is non-Riesz. We announce the following result.

Theorem. *Let $A(u)$ be a commuting family of real self-adjoint $n\times n$-matrices (u runs over the support of Φ), and $(\det A(u))^{-1/p}\Phi(u) \in L^1(\Omega)$ ($1 \le p \le \infty$). Then every nontrivial Hausdorff operator $\mathcal{H}_{\Phi,A}$ on $L^p(\mathbb{R}^n)$ is a non-Riesz operator (and in particular it is not a sum of quasinilpotent and compact operator).*

Corollary. *Every nontrivial Hausdorff operator on $L^p(\mathbb{R})$ is a non-Riesz operator (and in particular it is not a sum of quasinilpotent and compact operator).*

REFERENCES

1. *Liflyand, E.* Hausdorff operators on Hardy spaces. Eurasian Math. J., **4**, 101 – 141 (2013).
2. *Mirotin, A. R.* Addendum to "Boundedness of Hausdorff operators on Hardy spaces H^1 over locally compact groups". J. Math. Anal. Appl. **479**, 872 – 874 (2019).
3. *Mirotin, A. R.* The structure of normal Hausdorff operators on Lebesgue spaces. Forum Math. **32**, 111–119 (2020).

A. B. Morgulis (Rostov-na-Donu, Vladikavkaz, Russia)
morgulisandrey@gmail.com

EFFECT OF THE BOUNDARY CONDITIONS ON THE HYDRODYNAMIC STABILITY

Specifying the boundary values of the vector field of the fluid velocity stands as boundary conditions for the classical setting of the boundary value problems for the Navier-Stokes equations. Physically, it means that the fluid cannot slip alongside the boundary but generally can flow through it. The case of zero through-flow is the classical model of the rigid impermeable wall, while the general case is the simplest model of porous wall. However, the diversity of physical situations suggests the diversity of the boundary conditions. Also, in computational fluid dynamics, introducing artificial boundaries bearing so-called soft boundary conditions is often inevitable. In this communication, we discuss the influence of several kinds of boundary conditions on the permeable boundaries on the stability of fluid flows.

REFERENCES

1. *Ilin, K., Morgulis, A.* Instability of a two-dimensional viscous flow in an annulus with permeable walls to two-dimensional perturbations. Phys. Fluids. Vol. 27, 044107, 2015.

2. *Ilin K., Morgulis A.* On the stability of the Couette–Taylor flow between rotating porous cylinders with radial flow. /European Journal of Mechanics-B/Fluids. 2020. Vol. 80, pp. 174-186.

I. V. Morshneva (Rostov-on-Don, Russia)
ivmorshneva@sfedu.ru

BIFURCATING PERIODIC CONVECTIVE FLOWS IN A VERTICAL LAYER WITH MOVING BOUNDARIES

The problem of the occurrence of spatial secondary self-oscillating convection modes in an infinite vertical fluid layer with boundaries moving vertically with the same value, but opposite in direction, velocities is considered. The behavior of such system is described by a system of convection equations in Oberbeck-Boussinesq approximation.

The equations of motion have a steady state solution (basic mode) with a cubic velocity profile and a linear temperature distribution. It is known that for some parameter values, this solution becomes unstable with oscillations [1].

The occurrence of self-oscillations that appear with an oscillatory instability of the basic mode with respect to spatial perturbations is studied. The perturbation equations have a symmetry group $O(2) \times O(2)$ and the theory of Andronov-Hopf bifurcation in systems with symmetry is applicable [2]. Following the theory, as a governing parameter passes through its critical value the cycles can bifurcate from the equilibrium solution. These cycles correspond to self-oscillations such as horizontal traveling waves, oblique traveling waves and nonlinear superpositions of oblique traveling waves.

For various critical values of the parameters corresponding to an oscillatory instability, the analysis of the branching type and stability of the occurring spatial self-oscillating modes in a vertical layer with moving boundaries is made. In order to do this, the analytical expressions for the coefficients of the branching equations system are derived. For the self-oscillatory modes, the first two members of the series are written by degrees of the supercritical parameter.

REFERENCES

1. *Gershuni G. Z., Zhukhovitsky E. M., Nepomniachtchi A. A.* Stability of convective flows. Moscow, Nauka. 1989.

2. *Morshneva I. V.* The Andronov-Hopf Bifurcation in a Dynamical Systems with Symmetry and its applications in hydrodynamics. Rostov-on-Don, SFEDU. 2010.

E. Nakai (Mito, Japan)
eiichi.nakai.math@vc.ibaraki.ac.jp

INTEGRAL OPERATORS ON ORLICZ-MORREY AND WEAK ORLICZ-MORREY SPACES

Let $L^p(\mathbb{R}^n)$ and $\mathrm{w}L^p(\mathbb{R}^n)$ be the Lebesgue space and it weak version. For the Hardy-Littlewood maximal operator M, the following boundedness are well known:

$$\begin{aligned} M :& L^1(\mathbb{R}^n) \to \mathrm{w}L^1(\mathbb{R}^n), \\ M :& L^p(\mathbb{R}^n) \to L^p(\mathbb{R}^n) \quad (1 < p \le \infty), \\ M :& \mathrm{w}L^p(\mathbb{R}^n) \to \mathrm{w}L^p(\mathbb{R}^n) \quad (1 < p < \infty). \end{aligned}$$

In this talk, I will introduce some recent work on the boundedness of some integral operators on Orlicz-Morrey and weak Orlicz-Morrey spaces. The Orlicz-Morrey and weak Orlicz-Morrey spaces are contain Orlicz spaces, Morrey spaces, Lebesgue spaces and there weak versions as special cases. Then the boundedness on the Orlicz-Morrey and weak Orlicz-Morrey spaces imply the boundedness of these function spaces. For example, we have the boundedness of M from weak $\exp L^p(\mathbb{R}^n)$ to itself.

This talk is based on joint work with Ryutaro Arai, Ryota Kawasumi and Minglei Shi.

A. I. Nazarov (St. Petersburg, Russia)
al.il.nazarov@gmail.com

SPECTRAL ASYMPTOTICS FOR INTEGRO-DIFFERENTIAL OPERATORS GENERATED BY THE FBM-LIKE PROCESSES

We study the spectral problems for some integro-differential equations arising in the theory of Gaussian processes similar to the fractional Brownian motion. We generalize the method of Chigansky–Kleptsyna [1] and obtain the two-term eigenvalues asymptotics for such equations. Application to the small ball probabilities in L_2-norm is given.

The talk is based on the papers [2], [3] and on some recent results in preparation. The research is supported by the joint RFBR–DFG grant 20-51-12004.

R E F E R E N C E S

1. *Chigansky P. and Kleptsyna M.* Exact Asymptotics in Eigenproblems for Fractional Brownian Covariance Operators. Stochastic Processes and Their Applications. 2018. Vol. 128, No. 6, pp. 2007–2059.

2. *Nazarov A. I.* Spectral asymptotics for a class of integro-differential equations arising in the theory of fractional Gaussian processes. Communications in Contemporary Mathematics. 2020. DOI 10.1142/S0219199720500492, pp. 1–25.

3. *Nazarov A. I.* Some lemmata on the perturbation of the spectrum. Russian Journal of Mathematical Physics. 2020. Vol. 27, No. 3, pp. 378–381.

S. A. Nesterov (Vladikavkaz, Russia)
1079@list.ru

INVESTIGATION OF THE COEFFICIENT INVERSE PROBLEM OF THERMOELASTICITY BY THE ALGEBRAIZATION METHOD

Recently, structures in a high-temperature environment are increasingly made of functionally graded materials (FGM). FGM are composites with variable thermomechanical properties. The thermomechanical characteristics of the FGM after fabrication can be determined only by solving the coefficient inverse problem (CIP) of thermoelasticity. CIP of thermoelasticity are nonlinear problems. The most common approach to solving nonlinear CIPs is based on the use of iterative algorithms. At the same time, alternative non-iterative solution methods are being developed.

The paper investigates the CIP of thermoelasticity for inhomogeneous bodies based on the method of algebraization. We represent the thermomechanical characteristic in the form of a polynomial with unknown coefficients. The direct problem for an inhomogeneous rod and cylinder, both for mechanical and thermal loading, is solved by the Galerkin method in the space of Laplace transformants. When solving the problem by the Galerkin method, we calculate the determinant of the obtained system of algebraic equations. The determinant vanishes for some values of the Laplace transform parameter p. To find these values, we approximate the additional information measured at the boundary of a rod and cylinder at some points in time in the form of a linear combination of exponential functions. The exponents in the expansions are found by the Prony method. As a result of substitution of the values of p into the determinant, we obtain a system of nonlinear algebraic equations, the solution of which is sets of numbers. The criteria for selecting a suitable set are: 1) limited thermomechanical characteristics; 2) the minimum value of the residual functional.

Computational experiments to reconstruct thermomechanical characteristics were carried out. Only one of the characteristics was restored, the rest were assumed to be known. The reconstruction took place in the classes of linear and quadratic functions. Maximum error identification of monotone functions did not exceed 4 %.

The algebraization method allows only monotone functions to be reconstructed with high accuracy, but its implementation requires much less computer processing time than with the iterative approach.

P. V. Nikolenko, L. V. Novikova (Rostov-on-Don, Russia)
petr.v.nikolenko@gmail.com, lvnovikova@sfedu.ru

ABOUT SOME EXTREME PROBLEMS IN THE «INVESTMENT - CONSUMPTION» MODEL

The dynamics of capital-labor ratio in the investment-consumption model is described by the law

$$\dot{x} = sf(x) - \mu x,$$

where x — capital-labor ratio, f — production function, $f(x)$ — value produced by one worker per unit of time, s — the share of the value produced that is returned to production in the form of investment, μ — depreciation rate of production assets. If an additional amount is invested in the process S, which comes in the form of a financial flow $u(t)$, at that $0 \leqslant u(t) \leqslant p(x(t))$, where p — marginal ability to absorption of investments, then the dynamics of capital-labor ratio will take the form

$$\dot{x} = sf(x) - \mu x + u(t).$$

Let $x(0) = x_0$, $x(t_1) = x_1$, $x_1 > x_0$. Consider the tasks:

1. Define u so that the capital-labor ratio changes from x_0 up to x_1 for the minimum time.

2. The time t_1 is fixed, it is assumed that own investments $sf(x)$ are not enough to reach the capital-labor ratio of the value x_1 by the moment t_1. The question is, what is the minimum amount of S and in the form of what flow u it must be invested in order to satisfy the condition $x(t_1) = x_1$.

In both problems, the required function u can be written as a function of the argument x, and

$$u(x) = \begin{cases} 0, & \text{если } x \in [x_0', x_1'], \\ p(x), & \text{если } x \in \overline{[x_0', x_1']}, \end{cases}$$

where the bar denotes the complement of the set.

If $[x_0', x_1'] \subset (x_0, x_1)$, that $F(x_0') = F(x_1')$, where $sf(x) - \mu x$ denoted by $F(x)$. The values x_0', x_1' are determined from the relations:
for the task 1

$$\int\limits_{[x_0', x_1']} \frac{p(z)\,dz}{F(z) + p(z)} = S;$$

for the task 2

$$\int\limits_{[x_0', x_1']} \frac{dz}{F(z)} + \int\limits_{\overline{[x_0', x_1']}} \frac{dz}{F(z) + p(z)} = t_1.$$

L. G. Kurakin (Moscow, Vladikavkaz, Rostov-on-Don, Russia)
I. V. Ostrovskaya (Rostov-on-Don, Russia)
lgkurakin@sfedu.ru, ivostrovskaya@sfedu.ru

THE STABILITY PROBLEM OF A VORTEX QUADRUPOLE IN THE KIRCHHOFF MODEL

The stability problem of the stationary rotation of a vortex quadrupole in the Kirchhoff model is considered. The quadrupole consists of three point vortices of unit intensity located uniformly on a circle around a central vortex with arbitrary intensity $\varkappa$.

Analysis of the linearization matrix has shown that the investigated regime is exponentially unstable if $\varkappa > 1$ [1]. In [2] it was proved using the Chetaev method of coupling of first integrals, that in the case of $\varkappa < -3$ and $0 < \varkappa < 1$ the orbital stability in the exact nonlinear setting takes place. In the case $-3 \le \varkappa \le 0$ the stability problem needs the nonlinear analysis. It was found in [4] that for all values $\varkappa \in (-3, 0)$ in the stability problem, there are a resonance $1 : 1$ (diagonalizable case). Some other resonances up to the fourth order inclusive are found and investigated: double zero resonance (diagonalizable case), resonances 1:2 and 1:3, occurring with isolated values $\varkappa$. The stability of the equilibrium of the system reduced by one degree of freedom with the involvement of the terms in the Hamiltonian up to the fourth degree inclusive is proved for all $\varkappa \in (-3, 0)$. To the normal form of the Hamiltonian is constructed to terms of the fourth degree inclusively using the Birchhoff's algorithm. The stability of the reduced system equilibrium is proved with using the theorem of A. G. Sokol'sky [3] for the resonance 1:1 when taking into account terms up to the fourth order inclusive in the Hamiltonian.

The work of the first author was carried out in the frame of the Program No. 0147-2019-0001 (State Registration No. AAAA-A18-118022090056-0). The work of the second author was supported by Russian Foundation for Basic Research (Projects No. 20-55-10001).

REFERENCES

1. *Morikawa G.K., Swenson E.V.* Phys. Fluids. 1971. Vol. 14, No. 6, pp. 1058–1073.
2. *Kurakin, L. G., Ostrovskaya, I. V.* Rus. J. Nonlin. Dyn. 2019. Vol. 15, No. 4, pp. 533–542.
3. *Kunitsyn, A. N., Markeev, A. P.* Stability in resonance cases, in surveys in science and engineering. General Mechanics Series. Vol. 4. Moscow: VINITI, 1979, pp. 58–139.
4. *Kurakin, L. G., Ostrovskaya, I. V.* Regul. Chaot. Dyn. 2021. Vol. 26, No. 5. (accepted to print)

D. B. Rokhlin, G.A. Ougolnitsky (Rostov-on-Don, Russia)
dbrohlin@sfedu.ru

SOLO FTRL ALGORITHM FOR SETTING THE TRANSFER PRICE

Consider a firm producing and selling d goods and consisting of n production and m sales divisions. The manager seeks to stimulate the firm-optimal division behaviour by setting the internal prices $\lambda \in \mathbb{R}^d$ of goods (transfer prices). Division responses are defined as follows:

$$\widetilde{x}_i(\lambda) = \arg\max_{x_i \in X_i} (f_i(x_i) - \langle \lambda, x_i \rangle), \quad i = 1, \dots, m,$$
$$\widetilde{y}_i(\lambda) = \arg\max_{y_i \in Y_i} (\langle \lambda, y_i \rangle - g_i(y_i)), \quad i = 1, \dots, n,$$

where $f_i : X_i \mapsto \mathbb{R}_+$, $i = 1, \dots, m$ are the revenue functions of the sales divisions, and $g_i : Y_i \mapsto \mathbb{R}_+$, $i = 1, \dots, n$ are the cost functions of production divisions. In the present work these functions are assumed to be unknown.

In the static problem, where the functions f_i, g_i are fixed, under strong convexity and compactness assumptions, it is established that the SOLO FTRL algorithm [1], applied to the dual problem, provides estimates of order $T^{-1/4}$ in the number of T iterations for the sub-optimality gap of the total firm profit and for the imbalance between supply and demand. This algorithm uses information only about the reactions of divisions to current transfer prices:

$$\lambda_t = -\frac{\sum_{j=1}^{t-1} \Delta\widetilde{z}(\lambda_j)}{\sqrt{\sum_{j=1}^{t-1} \,|\Delta\widetilde{z}(\lambda_j)\,|^2}}, \quad \lambda_0 = 0; \quad \Delta\widetilde{z}(\lambda) := \sum_{i=1}^{n} \widetilde{y}_i(\lambda) - \sum_{i=1}^{m} \widetilde{x}_i(\lambda).$$

Similar results are obtained for the dynamic problem, where the functions f_i, g_i depend on the sequence of independent identically distributed random variables.

We present two computer experiments with one and two goods. In the static case the transfer prices and the difference between the supply and demand demonstrate fast stabilization. In the dynamic case the same quantities fluctuate around equilibrium values after a short transition phase.

REFERENCES

1. *Orabona F., Pál D.* Scale-free online learning. Theoretical Computer Science. 2018. Vol. 716, pp. 50–69.

D. S. Roshal, K. K. Fedorenko, S. B. Rochal (Rostov-on-Don, Russia)
M. Martin, V. Molle, S. Baghdiguian (Montpelie, France)
rochal.d@yandex.ru

ANALYSIS AND MODELING OF THE TOPOLOGY OF HEALTHY AND CANCEROUS EPITHELIAL MONOLAYERS

The geometry of cell monolayers depends not only on genetic control, but also on universal physical and topological laws. Here we analyze and model 3 fundamentally different types of epithelia: spherical non-proliferating epithelium of Ascidia, flat slowly dividing healthy and hyper-proliferative cancerous epithelia of the human cervix. We consider how the curvature of the epithelia surface, as well as the rate of cell division, affects the number of topological defects (cells, whose neighbors number differs from 6).

Thanks to the analysis of more than 140 photographs of spherical non-proliferative epithelium, we were the first to find complex topological defects (such as linear scars, pleats and nonlinear extended defects). Using computer modeling we show that the difference in cell sizes in Ascidia leads to a greater topological defectiveness than in spherical packings with structural units of the same size (colloidal crystals, viruses).

Recently, it has been shown that the cells distribution by the number of their neighbors is the same in the various flat proliferative epithelia. Here we show that hyperproliferation of cancer cells violates this fact and leads to random epithelia structures. We assume that the spread of cell sizes is the only parameter that controls the distribution of cells by the number of their neighbors in these monolayers. We test this hypothesis by considering randomly generated morphologically similar packings. We show that better ordering of healthy epithelial monolayers, as compared to cancerous ones, is associated with a lower proliferation rate and more efficient relaxation of mechanical stresses associated with cell division.

This work was supported by the RFBR grant № 19-32-90134.

REFERENCES

1. *Roshal D. S., Azzag K., Le Goff E., Rochal S. B., Baghdiguian S.* Crystal-like order and defects in metazoan epithelia with spherical geometry. Scientific Reports. 2020. Vol. 10, № 3, P. 1–11.

2. *Roshal D. S., Martin M., Fedorenko K. K., Molle V., Baghdiguian S., Rochal S. B.* Cancer cell hyper-proliferation disrupts the topological invariance of epithelial monolayers. Biorxiv. 2021. doi.org/10.1101/2021.08.12.455991

D. A. Ryabogin (Kent, USA)
ryabogin@math.kent.edu

A NEGATIVE ANSWER TO ULAM'S PROBLEM 19 FROM THE SCOTTISH BOOK

We give a negative answer to Ulam's Problem 19 from the Scottish Book asking, *is a solid of uniform density which will float in water in every position a sphere?* Assuming that the density of water is 1, we show that there exists a strictly convex body of revolution $K \subset \mathbb{R}^3$ of uniform density $\frac{1}{2}$, which is not a Euclidean ball, yet floats in equilibrium in every direction. We prove an analogous result in all dimensions $d \geq 3$.

REFERENCES

1. *Auerbach H.* Sur un probléme de M. Ulam concernant l'équilibre des corps flottants. Studia Mathematica, 1938, Vol. 7, No. 1, pp. 121–142.
2. *Bracho J., Montejano L., Oliveros D.* Carousels, Zindler curves and the floating body problem. Per. Mat. Hungarica, 2004, Vol. 49, No. 2, pp. 9–23.
3. *De La Vallée Poussin Ch. J.* Lecons De Mécanique Analytique, Vol II, 1925, Russian translation, Moscow, 1949.
4. *Falconer K. J.* Applications of a Result on Spherical Integration to the Theory of Convex Sets. Amer. Math. Monthly, 1983, Vol. 90, pp. 690–693.
5. *Florentin D. I., Schütt K., Werner E.M., Zhang N.* Convex floating bodies of Equilibrium, arXiv:2010.09006.
6. *Gilbert E. N.* How things float. Amer. Math. Monthly, 1991, Vol. 98, No. 3, pp. 201–216.
7. *Mauldin R. D.* The Scottish book, Mathematics from the Scottish Café with selected problems from the new Scottish book, Second Edition. Birkhäuser, 2015.
8. *Nazarov F., Ryabogin D., Zvavitch A.* An asymmetric convex body with maximal sections of constant volume. J. of AMS, 2014, Vol. 27, No. 1, pp. 43–68.
9. *Olovjanischnikoff S. P.* Ueber eine kennzeichnende Eigenschaft des Ellipsoides. Leningrad State Univ. Ann. (Uchen. Zap.), 1941, Vol. 83, pp. 113–128.
10. *Schneider R.* Functional equations connected with rotations and their geometric applications. L'Enseign. Math., 1970, Vol. 16, pp. 297–305.
11. *Ulam S. M.* A Collection of Mathematical Problems, Interscience, New York, 1960, p. 38.
12. *Várkonyi P. L.* Neutrally floating objects of density $\frac{1}{2}$ in three dimensions. Stud. Appl. Math., 2013, Vol. 130, No. 3, pp. 295–315.
13. *Wegner F.* Floating bodies of equilibrium. Stud. Appl. Math., 2003, Vol. 111, No. 2, pp. 167—183.
14. *Wegner F.* Floating bodies in equilibrium in $2D$, the tire track problem and electrons in a parabolic magnetic fields, 2007, arXiv:physics/0701241v3.
15. *Zhukovsky N. E.* Classical mechanics. Moscow, 1936.

N. A. Saifutdinova
(Rostov-on-Don, Russia, Don State Technical University)
saifut25@mail.ru

ON THE ECONOMIC INTERPRETATION OF THE INTRODUCTION OF STOCHASTIC PARAMETRS INTO SOME CLASSICAL OPTIMIZATION MODELS

In economic and mathematical modeling, one of the most popular tools for studying the relationship between factors of production and output is production functions. Different types of such functions allow us to take into account all the variety of conditions of the real economy. The most commonly used Cobb-Douglas production function or its analogues, for example, this function has the form: $F = K^{\alpha}L^{1-\alpha}$ herewith $0 \leq \alpha \leq 1$ Here K - the amount of capital used, L - the volume of labor resources, F - system output volume, α - coefficient of elasticity for the capital factor. In modern society, production technologies predominate, leading to a more efficient use of capital, which can undoubtedly be associated with an increase in elasticity for this factor, so we can say that the change in this indicator is a reflection of scientific and technological progress. We will assume that a change in production technology invariably leads to a change in α. In works [1],[2],[3],[4] various optimization models based on functions of this type are considered. In this case, the indicators become random variables and the expectation of F is optimized. In what situation does the elasticity indicator for the capital factor become a random variable? In a situation where the implemented technology is chosen randomly, namely, it is the result of a new business model, that is, a startup. This interpretation allows you to maximize the output of the system (or its profit), depending on the random choice of the implemented technology.

REFERENCES

1. *Pavlov, I.V.,Uglich, S.I.* Optimization of complex systems of quasilinear type with several independent priorities. Vestnik RGUPS. 3. 2017, pp. 140-145.
2. *Volosatova, T.A., Pavlov, I.V., Uglich, S.I.* Minimax problem for quasilinear complex systems with independent priorities. Vestnik RGUPS. 2. 2019, pp 149-154.
3. *Pavlov, I.V., Uglich, S.I.* On stationary points of the function of maximum arising in a quasilinear model with independent priorities. Vestnik RGUPS. 4. 2019, pp 143-148.
4. *Neumerzhitskaia, N.V., Uglich, S.I., Volosatova, T.A.* Sufficient conditions for the uniqueness of the maxima of the optimization problem in the framework of a stochastic model with priorities depending on one random variable. 2020, E3S Web of Conferences 224, 01014.

Samko S. (Rostov-on-Don, Russia; Faro, Portugal)
Karapetyants A. (Rostov-on-Don, Russia)
karapetyants@gmail.com

HADAMARD-BERGMAN CONVOLUTION OPERATORS AND GENERALIZED HOLDER SPACES

We introduce a convolution form, in terns of integration over the unit disc $\mathbb{D}$, for operators on functions f in $H(\mathbb{D})$, which correspond to Taylor expansion multipliers. We demonstrate advantages of the introduced integral representation in the study of mapping properties of such operators. In particular, we prove the Young-Hausdorff theorem for Bergman spaces in terms of integrability of the kernel of the convolution. This enables us to refer to the introduced convolutions as Hadamard-Bergman convolution. Another, more important, advantage is the study of mapping properties of a class of such operators in Holder type spaces of holomorphic functions, which in fact is hardly possible when the operator is defined just in terms of multipliers. Moreover, we show that for a class of fractional integral operators such a mapping between Holder spaces is onto. We pay a special attention to explicit integral representation of fractional integration and differentiation.

Y. Sawano (Tokyo, Japan)
yoshihiro-sawano@celery.ocn.ne.jp

LOCAL MUCKENHOUPT CLASS FOR VARIABLE EXPONENTS

Let $1 < p < \infty$ and let $\mathcal{Q}$ denote compact cubes whose edges are parallel to the coordinate axes. Recall that a locally integrable weight w is said to be an A_p-*weight*, if $0 < w < \infty$ almost everywhere, and

$$[w]_{A_p} \equiv \sup_{Q \in \mathcal{Q}} m_Q(w) m_Q(w^{-\frac{1}{p-1}})^{p-1} < \infty.$$

The quantity $[w]_{A_p}$ is referred to as the A_p-*constant*. We consider its local variant:

$$[w]_{A_p^{\rm loc}} \equiv \sup_{Q \in \mathcal{Q}, |Q| \le 1} m_Q(w) m_Q(w^{-\frac{1}{p-1}})^{p-1} < \infty.$$

This definition is due to Rychkov. We seek to pass his definition to the setting of variable exponent Lebesgue spaces.

Here not to get confused with some different definitions of weighted variable exponent Lebesgue spaces, we fix the notation. Herein, we use the following notation of variable exponents: Let $p(\cdot) : \mathbb{R}^n \to [1, \infty)$ be a measurable function, and let w be a weight. In other words, $w : \mathbb{R}^n \to [1, \infty)$ is a measurable function that is positive almost everywhere. Then the weighted variable Lebesgue space $L^{p(\cdot)}(w)$ collects all measurable functions f such that

$$\int_{\mathbb{R}^n} \left(\frac{|f(x)|}{\lambda} \right)^{p(x)} w(x) {\rm d}x < \infty$$

for some $\lambda > 0$. For $f \in L^{p(\cdot)}(w)$, the norm is defined by

$$\|f\|_{L^{p(\cdot)}(w)} \equiv \inf \left\{ \lambda > 0 \, : \, \int_{\mathbb{R}^n} \left(\frac{|f(x)|}{\lambda} \right)^{p(x)} w(x) {\rm d}x \le 1 \right\}.$$

If $w \equiv 1$, then $\| \cdot \|_{L^{p(\cdot)}(1)} = \| \cdot \|_{p(\cdot)}$ and $L^{p(\cdot)}(1) = L^{p(\cdot)}(\mathbb{R}^n)$. Thus, we have the ordinary variable Lebesgue space $L^{p(\cdot)}(\mathbb{R}^n)$.

What the speaker wants to convey in this talk is not the precise results but the skill that the local A_p-class gives us.

REFERENCES

1. *Rychkov, V.S.* Littlewood–Paley theory and function spaces with $A_p^{\rm loc}$ weights. Math. Nachr. (2001) **224**, 145–180.
2. *Nogayama T. and Sawano Y.* Local Muckenhoupt class for variable exponents, J. Inequal. Appl. (2021), № 70, 27 pp.

C. Sbordone (Naples, Italy)
sbordone@unina.it

ON THE EQUATION div $\mathbf{u} = f$ IN THE PLANE

The solvability for $\mathbf{u} \in W^{1,1}(Q_0; \mathbb{R}^2)$, $Q_0 = [0,1]^2$ of the equation

$$\text{div }\mathbf{u} = f \qquad f \in X \tag{1}$$

with

$$\|\mathbf{u}\|_Y \leq C\|f\|_X \tag{2}$$

where $X \subset L^1(Q_0)$ is a given Banach function space, is related to the gap between regularity of gradient of solutions $\varphi \in W_0^{1,1}(Q_0)$ and $\psi \in W_0^{1,1}(Q_0)$ of

$$\Delta\varphi = f$$

with

$$\|\nabla\varphi\|_Z \leq C\|f\|_X$$

and

$$\Delta\psi = \text{ div }\mathbf{u}$$

with

$$\|\nabla\psi\|_Y \leq C\|\mathbf{u}\|_Y$$

respectively.

Here, Y is the sharp r.i. target of X in Sobolev embedding $W^1X \hookrightarrow Y$ and $Y \subset Z \subset L^1$.

If (1), (2) has solution for any $f \in X$ then $Z = Y$ and

$$W^1(Y^\star) \hookrightarrow X^\star.$$

V. I. Semenov (Kaliningrad, Russia)
visemenov@rambler.ru

ASYMPTOTIC EXACT ESTIMATES FOR DISTORTION OF QUASICONFORMAL MAPPINGS OF A UNIT DISC

Theorem (A.Mori, 1956) Every $\mathbf{K}$ – quasiconformal mapping $\mathbf{f}$ of the unit disc $|\mathbf{z}| \leq \mathbf{1}$ onto itself with a condition $\mathbf{f(0) = 0}$ satisfies inequalities:

$$|\mathbf{f(z_1) - f(z_2)}| \leq \mathbf{16}|\mathbf{z_1 - z_2}|^{\mathbf{1/K}},$$

for every pair $\mathbf{z_1, z_2}$ and

$$|\mathbf{f(z)}| \leq \mathbf{4}|\mathbf{z}|^{\mathbf{1/K}}$$

for every $|\mathbf{z}| \leq \mathbf{1}$.

Remark . Constants $\mathbf{16, 4}$ are the best if $\mathbf{K} \to \infty$.

For a long time it was not clear the case when $\mathbf{K} \to \mathbf{1}$.

The basic hypothesis:

can we replace constants $\mathbf{16, 4}$ by $\mathbf{16^{1-1/K}}, \mathbf{4^{1-1/K}}$? Namely, these aspects are discussed in the talk.

I. A. Sheipak (Lomonosov Moscow State University, Moscow, Russia)
iasheip@yandex.ru

CONTINUOUS SPECTRUM IN THE STURM-LIOUVILLE PROBLEM WITH NON-COMPACT MULTIPLIER AS A WEIGHT

We consider Sturm–Liouville problem with a weight from the Sobolev space with a negative index of smoothness. If weight is a n-term self-similar non-compact multiplier the spectral problem for the string is equivalent to the spectral problem for the $(n-1)$-periodic Jacobian matrix. In the case $n = 3$ a complete description of the spectrum of the problem is given, and a criterion for the appearance of an eigenvalue in the gap of the continuous spectrum is obtained.

In the general situation $n \geqslant 3$, it is shown that the spectrum consists of $n-1$ segments of the continuous spectrum (some of which may touch) in the gaps between which there can be no more than $n-2$ eigenvalues (no more than one in each interval).

The work is supported by RFBR project No. 19-01-00240

M. A. Skopina (Saint Petersburg, Russia)
skopinama@gmail.com

WAVELET APPROXIMATION IN ORLICZ SPACES

Decompositions with respect to wavelet frames and frame-like systems are considered. Their approximation properties in Orlicz spaces are studied. In the case of wavelet frames, the approximation order in the sense of modular convergence is established for an arbitrary Orlicz space. Approximation order in the sense of Luxemburg norm is found for Orlicz spaces satisfying Δ_2- condition. Under assumption of Δ'- condition, the latter result is extended to the frame-like wavelet systems.

V. N. Rusev, A. V. Skorikov
(Gubkin University, Moscow, Russia)
vnrusev@yandex.ru, skorikov.a@gubkin.ru

THE MEAN RESIDUAL LIFE (MRL) OF THE WEIBULL-GNEDENKO DISTRIBUTION

The problem of finding sufficient conditions for the distribution of a random variable X with asymptotic exponential behavior of the distribution of the normalized residual lifetime X_t is considered (in other words, domain of residual life time attraction for X_t). Thus, it is necessary to find conditions for the distribution for which there is a continuous function $a(t)$ such that a weak convergence of the distribution tails takes place:

$$P\left(\frac{X-t}{a(t)} > x\,|X > t\right) \to e^{-x}, \quad t \to \infty.$$

For exponential distribution the article [1] gives the criterion describing the domain of attraction of the residual lifetime in terms of the coefficient of variation. Asymptotics of the mean residual life time and residual variance are obtained in article [2]. It gives possibility to find the domain of attraction of the limiting exponential distribution of the mean residual time in the class of Weibull - Gnedenko distributions.

This paper gives fairly simple sufficient conditions for distribution function to belong to the domain of residual life time attraction of exponential distribution.

Let $X \geq 0$ be a random variable with a distribution function F, $F(0) = 0$, and positive density $f(t) = F'(t)$ from class C^1, $\quad t \geq t_0$, with intensity $\lambda(t)$ also from class C^1.

Theorem 1. *If there is an asymptotic relation*

$$(\ln f(t))' \sim (\ln (1 - F(t)))' \quad (t \to +\infty)\,,$$

then the residual life time of the distribution F belongs to the domain of residual life times attraction of the exponential distribution.

REFERENCES

1. *Balkema A.A., Haan L.* Residual life time at great age. The Annals of probability. 1974. Vol. 2, No. 5, p. 792–804 .

1. *Rusev V., Skorikov A.* Asymptotic estimates of residual life time of the Gnedenko extreme - value distributions and applications. THE 5th International conference on stochastic methods (ICSM-5) Russia, Moscow, 23–27 november 2020 r. Kozyrev D.V. (eds) – Moscow : RUDN, 2020. – p. 360–364 .

S. N. Smirnov (Moscow, Russia)
s.n.smirnov@gmail.com

STRUCTURAL STABILITY OF FINANCIAL MARKET MODELS AND SUPERHEDGING PROBLEM

By the structural stability[1] we understand the fundamental property of a model, which means that the qualitative behaviour of the model is unaffected by small (in a certain sense) perturbations. From an economic point of view, such a qualitative behaviour of the model of the financial market is to admit no "arbitrage", in some sense to be made precise. In the framework of the deterministic market model in discrete time we introduce in [1] several relevant notions of "no arbitrage". Within this framework we consider discounted prices of n risky assets and formalise the uncertainty of price movements as follows: the vector of price increments lie in a priori given (non-void) compact sets $K_t(\cdot) \subseteq \mathbb{R}^n$, $t = 1, \ldots, N$, depending on price prehistory[2]; the trading constraints concern only risky assets and are described by a priori given sets depending on price prehistory, which are assumed to be convex and containing zero vector. According to our interpretation, $K_t(\cdot)$ reflects the agent's beliefs about price movements, which are naturally inexact; on the other hand, the trading constraints are supposed to be defined exactly. The concept of the structural stability in our context is formalised as follows: a specific "no arbitrage" property is unaffected by perturbations of $K_t(\cdot)$ that are sufficiently small with respect to the Pompeiu–Hausdorff metric. We argue that the structural stability is essential for the continuity property of superhedging price (see [2]) and for the uniform approximation propriety (see [3]). These results can be used to evaluate the accuracy of an approximation-based numerical method proposed in [2].

R E F E R E N C E S

1. *Smirnov S. N.*, A Guaranteed Deterministic Approach to Superhedging: No Arbitrage Properties of the Market. Automation and Remote Control. 2021. Vol. 82, No. 1, pp. 172–187.
2. *Smirnov S. N.*, A guaranteed deterministic approach to superhedging: Sensitivity of solutions of Bellman–Isaacs equations and numerical methods. Computational Mathematics and Modeling. 2020. Vol. 31, pp. 384–401.
3. *Smirnov S. N.*, A guaranteed deterministic approach to superhedging: Structural stability and approximation. Computational Mathematics and Modeling. 2021. 32, pp. 129–146.

[1] The term "structural stability" is borrowed from dynamical systems theory.
[2] The dot " $\cdot$ " indicates the variables representing the price prehistory.

I. Yu. Smirnova (Rostov-on-Don, Russia)
Smi_irina_dstu@mail.ru

WEIGHTED MIXED NORM HOLOMORPHIC SPACES DEFINED IN TERMS OF FOURIER COEFFICIENTS

Following the ideas of recent research in [1] we continue the study of new weighted spaces of holomorphic functions on the unit disc with the mixed norm defined in terms of conditions on Fourier coefficients of a function. Here we present a general approach for the weighted case. We study the boundedness of the weighted Bergman projection and discuss boundedness and compactness of Toeplitz operators with radial symbols. We also provide a characterization of functions in these spaces.

REFERENCES

1. *A. Karapetyants, S. Samko.* Mixed norm spaces of analytic functions as spaces of generalized fractional derivatives of functions in Hardy type spaces, Fract. Calc. Appl. Anal. no. 5 2017, 1106-1130.

V. V. Kazak (SFedU, Russia), N. N. Solokhin (DSTU, Russia)
vkazak136@gmail.com, nik2007.72@mail.ru

POINCARE'S BOUNDARY VALUE PROBLEM IN THE THEORY OF INFINITELY SMALL BENDINGS OF SURFACES

In the study of infinitesimal bendings of surfaces of positive curvature, various conditions are imposed on the edge of the surface during its deformation. The study of infinitesimal bendings of such surfaces is reduced to solving boundary value problems for generalized analytic functions. Consider a geometric condition of the form:

$$\alpha(\overline{U}, \bar{\ell}) + \beta(\overline{V}, \bar{L}) = \sigma \quad \text{on the} \quad \partial S, \tag{1}$$

where $\overline{U} = \overline{U}(x, y) \in C^{3,\mu}$, $\overline{V} = \overline{V}(x; y)$ — vectors of displacement and rotation of infinitesimal bending of a surface, vector fields $\bar{\ell}$, $\bar{L}$ and functions α, β, σ belong to the class C^{μ}, $0 < \mu < 1$. Study of this boundary condition for various α and β leads to the following boundary value problem:

$$\begin{cases} w_{\bar{z}} + B\bar{w} = 0, & z \in D, \\ \operatorname{Re}\{\overline{a(t)}w_t + \varepsilon b(t)w\} = \sigma, & t \in \partial D \end{cases} \tag{2}$$

For surfaces that are uniquely projected onto a plane, the following boundary value problem is obtained:

$$\begin{cases} w_{\bar{z}} + q_1 w_z + q_2 \bar{w}_{\bar{z}} = 0, & z \in D, \\ \operatorname{Re}\{\overline{a(t)}w_t + \varepsilon \overline{b(t)}w = \sigma, & t \in \partial D \end{cases} \tag{3}$$

These problems are reduced to a system of integral equations, which can be written in the form $\hat{U} = \varepsilon T\hat{U} + \sigma$. The study of this system makes it possible to judge the nature of the rigidity of the surface, subject to the mixed boundary condition at the edge (1).

R E F E R E N C E S

1. *Fomenko V.T.* On the quasi-correctness of external constraints in the theory of infinitesimal bendings. Siberian Mathematical Journal. 1974. T.XV, No. 1. P.152-161.
2. *Vinogradov V.S.* On a method for solving the Poincare problem for analytic functions. Reports of the Academy of Sciences of the USSR. 1960. P. 17-19.
3. *Kazak V.V., Solokhin N.N.* On the quasi-correctness of the mixed boundary condition for one class of surfaces. Modern problems of mathematics and mechanics. T.VI, issue 2. Publishing house of Moscow University. 2011. P.212-216.

I. Pavlov, A. Danekyants, N. Neumerzhitskaia, I. Tsvetkova
(DSTU, Rostov-on-Don, Russia)
pavloviv@mail.ru

INTERPOLATING SIGNED DEFLATORS

Consider a stochastic basis $\big(\Omega, F=(\mathcal{F}_k)_{k=0}^K, P\big)$, where Ω be a set, $F=(\mathcal{F}_k)_{k=0}^K$ be a strictly increasing filtration, $\mathcal{F}_0=\{\Omega,\emptyset\}$, $K<\infty$, $\mathcal{F}_K$ be finite, and P be a probability on $\mathcal{F}_K$. We assume that the probability measure P loads all non-empty subsets from $\mathcal{F}_k$, $0\le k\le K$. Let $Z=(Z_k,\mathcal{F}_k)_{k=0}^K$ be an adabted process that can take any real values. A martingale $D=(D_k,\mathcal{F}_k,P)_{k=0}^K$ is said a signed deflator of the process Z if $D_0{=}1$ and the process $DZ=(D_kZ_k,\mathcal{F}_k,P)_{k=0}^K$ is a martingale.

In the paper [1], a procedure is proposed for interpolating the process Z using the deflator D, which leads to the uniqueness of the deflator. In the case of the existence of martingale measures for the process Z, this procedure coincides with interpolation with respect to the martingale measure (c.f. [2]). In this context we introduce so-called special Haar uniqueness property (SHUP) for deflators (see [1]).

We use in the sequel the following system of notations. Let A be an atom in $\mathcal{F}_k$, B_i $(i=1,2,\ldots,m)$ be atoms in $\mathcal{F}_{k+1}$, $A=B_1+B_2+\cdots+B_m$, $a:=Z_k|_A$, $b_i:=Z_{k+1}|_{B_i}$, $p_i:=P(B_i)$, $d_i:=D_{k+1}|_{B_i}$. A signed deflator D of the process Z is said admissible if $\forall 0\le k<K$, for all atom $A\in\mathcal{F}_k$ and for all non-empty subset $I\subset\{1,2,\ldots,m\}$ we have $\sum\limits_{i\in I}p_id_i\neq 0$.

Theorem 1. *Let $\forall k: 0\le k<K$ and for all atom $A\in\mathcal{F}_k$ we have $m\ge 3$. If there exists an admissible signed deflator D satisfying SHUP, then the numbers $a,b_1,\ldots,b_m$ are different.*

Theorem 2. *Let $\forall k: 0\le k<K$ and for all atom $A\in\mathcal{F}_k$ we have $m\ge 4$ and the numbers $a,b_1,\ldots,b_m$ be different. Then there exists an admissible signed deflator D satisfying SHUP.*

It is easy to see that if in Theorem 2 $m=3$ for all atom $A\in\mathcal{F}_k$, then all admissible signed deflater D satisfies SHUP.

This research is supported by the RFBR (project 19-01-00451).

REFERENCES

1. *Pavlov I. V., Danekyants A. G., Neumerzhitskaia N. V., Tsvetkova I. V.* Signed interpolating deflators and Haar uniqueness properties. Global and stochastic analysis. 2021. Vol. 8, No. 2, pp. 67–75.

2. *Bogacheva, M. N., Pavlov I. V.* Haar extensions of arbitrage-free financial markets to markets that are complete and arbitrage-free. 2002. Russian Math. Surveys. Vol. 57, No. 3, pp. 581–583.

B. G. Vakulov[a], Yu. E. Drobotov[b]
(Rostov-on-Don, Russia)
[a]bvak1961@bk.ru, [b]yu.e.drobotov@yandex.ru

HYPERSINGULAR INTEGRALS ON METRICAL SETS IN THE GENERALIZED VARIABLE HÖLDER SPACES WITH WEIGHTS [1]

Let Ω be an open bounded set of a metrical space $(\mathbb{X}, d)$, such that all balls

$$B(x,r) = \{y \in \mathfrak{S} : d(x,y) < r\}$$

are measurable and their meausres satisfy the growing condition

$$\mu\,[B(x,r)] \leq Kr^N \quad \text{and} \quad r \to 0, \quad K > 0,$$

with $N > 0$ not necessarily integer.

The presented research is on hypersingular integrals of the form

$$D^{\alpha(\cdot)}f(x) = \lim_{\varepsilon\to 0} \int\limits_{y\in\Omega:d(x,y)>\varepsilon} \frac{f(y)-f(x)}{d^{N+\alpha(x)}(x,y)}d\mu(y), \quad x \in \Omega. \tag{1}$$

These operators were studied in [1] for constant α in the classical Hölder spaces $H^\lambda(\mathbb{X})$. In [2], the case of variable $\alpha(x)$ was considered for the special weight $\mathfrak{Re}[\alpha(x)]$. In [3], variable-order hypersingular integrals were proved to be bounded from the Hölder space with variable exponent $\lambda(x)$ into the space with „worse“ exponent $\lambda(x) - \alpha(x)$ with $\alpha(x) < \lambda(x)$.

In the talk, (1) is considered in the function space $H^{\omega(\cdot)}(\Omega, w)$ defined by the following condition on the local coninuity modulus of wf:

$$\Omega_d(wf,x,t) := \sup_{y\in\Omega:d(x,y)\leq t} |(wf)(x) - (wf)(y)| \leq A\omega(x,t), \quad A, t > 0.$$

The case of exponential weight $w(x)$ is studied, and the boundedness theorems for such spaces are proved.

REFERENCES

1. *Vakulov B. G.* Spherical operators of potential type in weighted Hölder spaces of variable order. – Vladikavkaz. Mat. Zh. – 2005 – Vol. 7, № 2. – 26–40.

2. *Garcia-Cuerva J., Gatto A. E.* Boundedness properties if fractional integral operators associated to non-doubling measures. – Studia Math. – 2004. – Vol. 162. – 245–261.

3. *Samko N. G., Samko S. G., Vakulov B. G.* Fractional integrals and hypersingular integrals in variable order Hölder spaces on homogeneous spaces. – Journal of Function Spaces and Applications. – 2010. – Vol. 8. – № 2. – 215–244.

[1]The reported study was funded by RFBR and TUBITAK according to the research project №20-51-46003.

N. P. Volchkova, Vit. V. Volchkov, N. A. Ischenko (Donetsk)
volna936@gmail.com, nataandRap@mail.ru

ON FUNCTIONS WITH VANISHING BALL MEANS

We study continuous functions on $\mathbb{R}^n$ that have zero integrals over all balls in $\mathbb{R}^n$ that are congruent to the unit ball with respect to a special group of Möbius transformations. An exact condition is found under which the functions of the class under consideration, extended in a corresponding way at the point at infinity, have a similar property on the Möbius space $\overline{\mathbb{R}}^n$.

For any $a \in \mathbb{R}^n$, we define the Möbius transformation

$$\mathcal{T}_a \in \mathrm{GM}(\overline{\mathbb{R}}^n) \qquad \text{by} \qquad \mathcal{T}_a(x) = a - (1 + |a|^2)(x^* + a)^*.$$

Let $\mathcal{P}$ be a set in $\mathrm{GM}(\overline{\mathbb{R}}^n)$ consisting of all mappings $\mathcal{T}_a$, inversion $x \to x^*$ and orthogonal transformations $\tau \in \mathrm{O}(n)$. We denote by G the subgroup in $\mathrm{GM}(\overline{\mathbb{R}}^n)$ generated by the set $\mathcal{P}$.

Theorem 1. *1) Assume that $f \in C(\mathbb{R}^n)$, $f(x) = o(|x|^{1-n})$ for $x \to \infty$, and*

$$\int\limits_{g(B)} f(x)dx = 0$$

for every $g \in G : g(B) \subset \mathbb{R}^n$. Then

$$\int\limits_{g(B)} f(x)dx = 0$$

for all $g \in G$.

2) There exist a function $f \in C(\mathbb{R}^n)$ such that

$$\int\limits_{g(B)} f(x)dx = 0$$

for all $g \in G : g(B) \subset \mathbb{R}^n$, $f(x) = O(|x|^{1-n})$ for $x \to \infty$, and the integral

$$\int\limits_{g(B)} f(x)dx$$

diverges for some $g \in G$.

The second assertion of Theorem 1 shows that the condition $f(x) = o(|x|^{1-n})$, $x \to \infty$, is close to final. For the general theory of a class functions with zero integrals over balls of fixed radius and its generalizations, see [1]–[3].

REFERENCES

1. *Volchkov V. V.* Integral Geometry and Convolution Equations. Kluwer Acad. Publ. 2003.
2. *Volchkov V. V., Volchkov Vit. V.* Harmonic Analysis of Mean Periodic Functions on Symmetric Spaces and the Heisenberg Group. Springer. 2009.
3. *Volchkov V. V., Volchkov Vit. V.* Offbeat Integral Geometry on Symmetric Spaces. Birkhäuser. 2013.

V. V. Volchkov, Vit. V. Volchkov (Donetsk)
valeriyvolchkov@gmail.com, volna936@gmail.com

AN ANALOG OF THE SERRIN THEOREM FOR HARMONIC FUNCTIONS

In 1971, J. Serrin (see [1]) initiated the studying of overdetermined boundary value problems for elliptic partial differential equations. We establish the property of radial symmetry for harmonic functions in unbounded domains on the complex plane. In contrast to predecessors' papers, in the proofs we use some results from theory of H_p spaces and boundary properties of conformal mappings.

Let Γ be a closed smooth Jordan curve in the complex plane $\mathbb{C}$, G be a bounded domain in $\mathbb{C}$ with the boundary Γ, $\overline{G} = G \cup \Gamma$, $\mu(E)$ be the measure (length) of the set $E \subset \Gamma$. For a function f which is continuous on the circle $\gamma_r = \{z \in \mathbb{C} : |z| = r\}$ we set $M_r(f) = \int_{\gamma_r} |f(z)||dz|$.

Theorem 1. *Let there exists a function $f \in C(\mathbb{C} \setminus G)$ which is harmonic in $\mathbb{C} \setminus \overline{G}$ and satisfies the following conditions: 1) $f = 0$ on Γ; 2) $\dfrac{\partial f}{\partial n} = 1$ μ-almost everywhere in Γ; 3) $\varliminf\limits_{r\to\infty} \dfrac{1}{r^3} M_r(f) = 0$. Then the domain G is a disk, wherein $f(z) = R \ln \frac{|z-z_0|}{R}$, where z_0, R are the center and the radius of the disk G.*

The necessity of conditions 1), 2) and exactness of condition 3) in Theorem 1 is justified by

Theorem 2. *There exists a bounded centrally symmetric domain $G \subset \mathbb{C}$, different from a circle, with the smooth Jordan boundary Γ and functions f_1, f_2, f_3, which are continuous in $\mathbb{C} \setminus G$ and harmonic in $\mathbb{C} \setminus \overline{G}$ such that: 1) f_1 satisfies conditions 1) and 3) in Theorem 1; 2) f_2 satisfies condition 3) of Theorem 1 and $\frac{\partial f_2}{\partial n} = 1$ everywhere in Γ; 3) $f_3 = 0$ and $\dfrac{\partial f_3}{\partial n} = 1$ everywhere in Γ, wherein $f_3(z) = O\left(|z|^2\right)$ for $z \to \infty$. In particular, $M_r(f_3) = O\left(r^3\right)$ for $r \to \infty$.*

On the application of results of this type to questions of integral geometry, see [1]–[3] and the bibliography available there.

REFERENCES

1. *Volchkov V. V.* Integral Geometry and Convolution Equations. Kluwer Acad. Publ. 2003.
2. *Volchkov V. V., Volchkov Vit. V.* Harmonic Analysis of Mean Periodic Functions on Symmetric Spaces and the Heisenberg Group. Springer. 2009.
3. *Volchkov V. V., Volchkov Vit. V.* Offbeat Integral Geometry on Symmetric Spaces. Birkhäuser. 2013.

I. Pavlov, N. Neumerzhitskaia, S. Uglich, T. Volosatova
(DSTU, Rostov-on-Don, Russia)
pavloviv@mail.ru

STRICT CONCAVITY OF SOME RANDOMIZED OBJECTIVE FUNCTIONS AND APPLICATION TO OPTIMIZATION PROBLEMS

In this work, we continue research related to the study of the maximum points of the objective function arising in a system with random priorities (see [1-2]). This system consists of several institutions and "optimizer" interested in the successful functioning of the system. At the same time, expert assessments should be implemented in the selection of independent or dependent priorities with different distributions.

Let $\alpha_1, \alpha_2, \alpha_3$ be random variables (r.v.) on a probability space $(\Omega.\mathcal{F}, P)$ such that P-almost surely (a.s.) $0 < \alpha_i < 1, \forall\, i = 1, 2, 3$. We call these r.v. priorities. Consider the function

$$F\left(u_1, u_2, u_3\right) := E\left(u_i^{\alpha_1} u_i^{\alpha_2} u_i^{\alpha_3}\right), \quad \text{where} \quad u_3 = -c_1 u_1 - c_2 u_2 + c_3 \left(c_i > 0, \forall i = 1, 2, 3\right).$$

The objective function $F(u_1, u_2)$ is defined and continuous on the set $D : u_i \geq 0$ $(i = 1, 2)$, $c_1 u_1 + c_2 u_2 \leq c_3$. On the boundary of D we have $F = 0$. The function F is infinitely differentiable on the interior D^0 of D and $F > 0$ on D^0. It is clear that on D^0 there is a stationary point (at which the function F has a global maximum). We are investigating whether it is only one.

Lemma. *If P-a.s. $\alpha_1 + \alpha_2 \leq 1$, $\alpha_1 + \alpha_3 \leq 1$ and $\alpha_2 + \alpha_3 \leq 1$, then the function F is concave on D^0. If, in addition to this, one of the conditions $P(\alpha_1 + \alpha_2 < 1) > 0$, $P(\alpha_1 + \alpha_3 < 1) > 0$ or $P(\alpha_2 + \alpha_3 < 1) > 0$ is fulfilled, then the function F is strictly concave on D^0.*

Theorem. *If the conditions of the Lemma, which ensure the strict concavity of the function F, are satisfied, then the function F has exactly one local (and simultaneously global) maximum point in D^0.*

This theorem essentially strengthens one of the sufficient conditions for the uniqueness of the maximum contained in [1].

This research is supported by the RFBR (project 19-01-00451).

REFERENCES

1. *Neumerzhitskaia N. V., Uglich S. I., Volosatova T. A.* Sufficient conditions for the uniqueness of the maxima of the optimization problem in the framework of a stochastic model with priorities depending on one random variable. E3S Web of Conferences. 2020. Vol. 224, 01014.

2. *Pavlov I. V., Uglich S. I.* Optimization of complex systems of quasilinear type with several independent priorities. 2017. Vestnik RGUPS. No. 3, pp. 140–145.

P. A. Zelenchuk, V. G. Tsybulin
(SFDU, Rostov-on-Don, Russia),
T. D. Ha (SFEDU, Hanoi, Vietnam)
zelenchukpavel@mail.ru

EFFECTS OF SPATIAL HETEROGENITY IN MATHEMATICAL MODELS OF THE PREDATOR-PREY SYSTEM

Modern ecology and mathematical biology are based on mathematical modeling. An urgent problem is the population system under conditions of heterogeneity of the habitat in the presence of diffusion–advective processes. We study the effects of spatial distribution in the predator–prey system caused by the heterogeneity of the prey resource.

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[k_1\frac{\partial u}{\partial x} - u\left(\alpha\frac{\partial p}{\partial x} - \beta_1\frac{\partial v}{\partial x}\right)\right] + u\left[a_1 u\left(1-\frac{u}{p}\right) - f_1\right],$$
$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\left[k_2\frac{\partial v}{\partial x} - \beta_2 v\frac{\partial u}{\partial x}\right] + v\left[-a_2 + f_2\right].$$

Here $p(x)$ is the resource function (the capacity of the environment for the prey), a_1 – is the prey's intrinsic growth parameter, a_2 – is the predator's mortality parameter. The terms with the coefficients k_i and α, β_i characterize the diffusion and migration processes respectively. The predation effect (terms of f_i) is represented by the functional response of Holling II type

$$f_1 = \frac{b_1 v}{1+cu}, \quad f_2 = \frac{b_2 u}{1+cu}$$

We study analytically and numerically the distribution of species for different variants of the description of local interaction. We analyze the dependence of the coefficients b_i, c on the inhomogeneous function of the resource $p(x)$. Several paradoxical scenarios of spatial dynamics are observed. We propose the trophic function corresponding to the correlation distribution of species

$$f_1 = \frac{b_1 v p(x)}{p(x)+cu}, \quad f_2 = \frac{b_2 u}{p(x)+cu}.$$

It established its robustness through computer experiments with a one-dimensional problem on the heterogeneous ring. We implement the method of lines with staggered grids and a special approximation of nonlinear terms.

A. Yu. Savin, K. N. Zhuikov (Moscow, Russia)
antonsavin@mail.ru, zhuykovcon@gmail.com

ON A GENERALIZATION OF THE MELROSE'S ETA-INVARIANT

The η-invariant was introduced by Atiyah, Patodi and Singer in their famous work [1] in 1975. It plays a fundamental role in elliptic theory and appears in many index formulas.

An important generalization was given by Melrose [2] in 1995. For families of parameter-dependent pseudodifferential operators (PDOs) $D(p)$, $p \in \mathbb{R}$, the η-invariant was defined as a special regularization of the winding number. The latter has the form

$$\frac{1}{2\pi i}\int_{\mathbb{R}} \operatorname{tr}\left(D^{-1}(p)\frac{dD(p)}{dp}\right)dp,$$

where $D(p)$ is elliptic and invertible for all $p \in \mathbb{R}$.

The main goal of the current work is to define the η-invariant for families

$$D(p) = \sum_{k\in\mathbb{Z}} D_k(p)e^{2\pi ikp}\colon C^\infty(X) \longrightarrow C^\infty(X), \tag{1}$$

where X is a smooth closed manifold, $D_k(p)$ is a family of parameter-dependent PDOs on X. Such families appear, for instance, in case of elliptic PDOs with shifts on a manifold with cylindrical ends (see [3]).

To define the η-invariant of families (1), using Melrose's approach, we introduce certain regularizations for the trace tr and the integral. Further, we establish main properties of the η-invariant. Namely, the η-invariant satisfies logarithmic property and generalizes the Melrose's η-invariant, i.e coincides with the latter in case of parameter-dependent PDOs. Finally, we present a formula for variation of the η-invariant as the family changes.

The current work is supported by the Ministry of Science and Higher Education of the Russian Federation: agreement no. 075-03-2020-223/3 (FSSF-2020-0018).

R E F E R E N C E S

1. *Atiyah M., Patodi V., Singer I.* Spectral asymmetry and Riemannian geometry I. Math. Proc. Cambridge Philos. Soc. 1975. Vol. 77, pp. 43–69.

2. *Melrose R.* The eta invariant and families of pseudodifferential operators. Math. Research Letters. 1995. Vol. 2, No. 5, pp. 541–561.

3. *Savin A. Yu., Sternin B. Yu.* Elliptic G-operators on manifolds with isolated singularities. J. Math. Sci. (N.Y.). 2018. Vol. 233, No 6, pp. 930–948.