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Preface

A. Karapetyants[1] · V. Kravchenko[2] · E. Liflyand[3]

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What is presented to the readers is a collection of works dedicated to the 80-th birthday of Stefan Samko, a recognized mathematician, mentor, colleague or/and just a friend of the contributing authors and managing editors. Here we introduce both Stefan whose anniversary is celebrated and the works of his colleagues dedicated to this jubilee. Short list of highlights: Stefan Samko has authored about 300 research papers and 5 monographs; since 1978 he has been organizing conferences and sessions of conferences in Russia (SU), USA, Japan, France, Italy, Turkey, UK, Portugal and Brazil; he travels a lot, participating as invited speaker in many conferences worldwide; Stefan Samko is a member of Editorial Boards of 13 Scientific international Journals; he was a coordinator of several major Russian and European grants, including INTAS project which united four teams from Finland, Portugal, Georgia, Azerbaijan. Now, more details and personal identity.

1 Stefan's Samko Professional Career: A Brief Overview

Stefan Samko was born on March 28, 1941 in Rostov-on-Don in a family of university professors Grigory and Valentina Samko. In 1958–1964, Stefan was a student of the Physics and Mathematics Faculty of Rostov State University (now the Southern Federal University, https://sfedu.ru/).

In 1964, Stefan Samko became a PhD student of the same university at the very influential scientific school of Professor F.D. Gakhov in the field of boundary value

✉ A. Karapetyants
karapetyants@gmail.com

V. Kravchenko
vkravchenko@math.cinvestav.edu.mx

E. Liflyand
liflyand@gmail.com

1 Institute of Mathematics, Mechanics and Computer Sciences and Regional Mathematical Center, Southern Federal University, Rostov-on-Don, Russia

2 Department of Mathematics, Cinvestav, Santiago de Queretaro, Mexico

3 Department of Mathematics, Bar-Ilan University, Ramat Gan, Israel

problems and singular integral equations. Stefan Samko's PhD research was devoted to the study of the solvability of some integral equations of the first kind, such as the generalized Abel integral equation, which included one-sided forms of fractional integration. This naturally led Stefan Samko to the deep exploration of fractional calculus, beginning with the one-dimensional theory and followed by the multidimensional integrodifferentiation.

His interest in Sobolev-type spaces of fractional smoothness naturally arising in multidimensional fractional calculus inspired interactions with researchers from the Steklov Mathematical Institute. He was an active participant in the well-known seminar, ruled by Academician S.M. Nikolsky, at the Steklov Institute, and he defended the second scientific degree (Doctor of Science) in this institution. This second degree dissertation was devoted to the development of the theory of hypersingular integrals.

His impact in the field of one-dimensional and multidimensional fractional calculus is reflected in his monograph S. Samko "Hypersingular integrals and their generalizations" (2001), and encyclopedic type book "Fractional Integrals and Derivatives", written together with Professors O.I. Marichev and A.A. Kilbas (1993). This encyclopedic book became a handbook for numerous researchers, and the number of references to it amounts to thousands.

Stefan Samko is well known for his collaboration with many people worldwide on different subjects exploring new areas. He is also a good friend who can carry on a friendship throughout the lifetime, and the research tandem N. Karapetiants–S. Samko showed an example of a lifelong sincere friendship and fruitful scientific work in the field of singular integral equstions with Carleman shift and later with a general approach to equations with the so called involutive operator. Part of their joint research appeared as the book by N. Karapetiants and S. Samko "Equations with Involutive Operators" (2001).

In 1992–1993 S. Samko was a Fulbright Professor at the University of New Haven, USA. One of the breakthrough results at that time, obtained jointly with Professors E.-R. Love and B. Ross, was an analogue of the Weierstrass example: they constructed an example of a function having all the derivatives of order less than one at each point, though having nowhere a first derivative. Also, S. Samko and B. Ross introduced and studied fractional integrals of the Riemann–Liouville type with variable order, that may be considered as the first steps of Professor Samko toward the area of research known as variable exponent analysis major to him for the last two and half decades.

Stefan Samko became one of the pioneers in the field of variable exponent analysis; he and his followers and students have succeeded in obtaining a number of important results in this area. This period of his tenure is already associated with his work as Professor at the University of Algarve, Portugal. Among other results he proved the Sobolev theorem in variable exponent Lebesgue spaces (1998) and showed the density of smooth functions in Sobolev spaces with variable exponent (1999). His interests in this modern field of mathematics may be described as operators of harmonic analysis in various general spaces of functions with non-standard growth. An essential part of his recent research in the field of nonstandard spaces and integral operators appeared in 2016 in the two-volume monograph written by S. Samko together with V. Kokilashvili, A. Meskhi, and H. Rafeiro: "Integral operators in non-standard function spaces, Volume 1: Variable exponent Lebesgue and Amalgam Spaces"; and "Inte-

gral operators in non-standard function spaces, Volume 2: Variable Exponent Hölder, Morrey—Campanato and Grand Spaces".

Stefan Samko's scientific interests are very broad, and he is constantly digging for new areas of research. His recent work has also been devoted to the theory of functions and operators in complex analysis, while his general encyclopedic knowledge and skills always allow him to consider problems in new perspectives and find original methods and solutions. Some of his recent work in this area (together with one of the authors) is related to classes of operators in cokplex analysis and Hölder-type holomorphic spaces. Some breakthrough results are related to recently introduced Hadamard–Bergman convolution operators, which is a form of convolution in terms of integration over the unit disc for operators on holomorphic functions that correspond to the Taylor multipliers. In such a form the authors demonstrate many advantages, including efficient study of such operators within the scale of Hölder spaces, which is hardly possible, in fact, when operator is defined only in terms of multipliers.

He had never forgotten the path to direct results and methods of harmonic analysis. For example, his survey paper on Wiener algebras (joint with Trigub and one of the authors) has almost immediately become a very important source with numerous references.

Stefan Samko is known as a brilliant teacher. He developed and taught several major courses while working in the Rostov State University and then in the University of Algarve, Portugal. One of the authors of this article had the good fortune of listening to Professor Samko's courses and can give evidence that Samko's lectures were always extremely exciting for students, the material was presented with ease, but at the same time with exceptional mathematical accuracy of formulation and proofs. Stefan Samko brought up many students and followers and created a scientific school on a global scale. He is a scientific advisor of 21 defended Ph.D. theses and 1 Doctor of Science thesis. His students are now working in various countries worldwide, and Stefan Samko continues to work and/or to stay in touch with many of them, always sharing knowledge and his invaluable experience.

The collection of works we are going to overview is a part of the tribute to his achievements from many colleagues and (simultaneously!) friends. It will not be surprising if Stefan not only will be pleased with reading these but generate certain new ideas, which he will gladly share with the authors.

We have made an attempt to briefly describe the highlights of Professor Samko's career. We used Editorial materials from the journal Fractional Calculus and Applied Analysis entitled "Anniversary of Prof. S.G. Samko, FC Events (FCAA-Volume 24-2-2021). Let us also refer to the recent paper by A. Almeida, Z. Kusraeva and Humberto Rafeiro, "Professor Stefan G. Samko Research: a Decade Retrospective" in Journal of Mathematical Sciences (2022) in which the authors present an overview of the most recent achievements of Professor Samko and his collaborators. Finally, we mentioned the overview paper by V. Kokilashvili "Stefan G. Samko—Mathematician, Teacher and Man" in the Stefan Samko Anniversary Volume entitled Advances in Harmonic Analysis and Operator Theory (2013) in Operator Theory: Advances and Applications book series (OT, volume 229).

2 Contributions to the Thematic Issue

From the nineteenth century to nowadays, through the very fruitful twentieth century, harmonic (Fourier) analysis has worked its way up from the study of classical problems of convergence of trigonometric expansions and integrals to a broad area where various spaces, new and old, are thoroughly investigated, numerous operators and their actions on these spaces are considered, and tools from mathematics as a whole are involved. Stefan Samko has contributed to all of these directions and such is the presented collection of works of his friends, collaborators and colleagues intended to pay a tribute to his achievements. In a somewhat artificial manner, we split our overview into certain topical parts. Needless to say that this division, as well as any other similar division, is very relative, first of all because of the unity of Fourier analysis (similar to the unity of mathematics in general) and various interrelations of these parts, very often seen even in terminology.

Carleson's solution of Luzin's problem got a just recognition as the main achievement in the classical Fourier analysis in the twentieth century. The problems concerned with this result continue to be in the focus of modern research. The paper "An analogy of the Carleson–Hunt theorem with respect to Vilenkin systems" by L.-E. Persson, F. Schipp, G. Tephnadze and F. Weisz is a good example of such a continuing interest in these problems. The title tells us what this study is devoted to. In particular, the theory of martingales is used and a new and shorter proof of the almost everywhere convergence of Vilenkin–Fourier series of $f \in L_p(G_m)$ for $p > 1$ in case the Vilenkin system is bounded is given. Moreover, the sharpness is proved by stating an analogy of the Kolmogorov theorem for $p = 1$ and constructing a function $f \in L_1(G_m)$ such that the partial sums with respect to Vilenkin systems diverge everywhere.

Interpolation of spaces and operators (and its counterpart extrapolation) is one of the central parts and tools in various topics of analysis. Many fabulous results in harmonic analysis are obtained by means of interpolation. The collection in question contains two works in this subject. In "Revisiting Yano extrapolation theory" by Elona Agora, Jorge Antezana, Sergi Baena-Miret, María J. Carro, a pointwise estimate for the decreasing rearrangement of Tf, where T is any sublinear operator satisfying the weak-type boundedness

$$T : L^{p,1}(\mu) \to L^{p,\infty}(\nu), \quad \forall p : 1 < p_0 < p \le p_1 < \infty,$$

with norm controlled by $C\varphi\left(\left[p_0^{-1} - p^{-1}\right]^{-1}\right)$ and φ satisfying some admissibility conditions, is proven. In particular, the obtained estimate allows one to derive extensions of Yano extrapolation results. In "Interpolation of Generalized Gamma Spaces in a Critical Case" by Irshaad Ahmed, Alberto Fiorenza and Maria Rosaria Formica, some interpolation formulae for generalized gamma spaces with double weights in a critical case are established. The used approach is based on identifying generalized gamma spaces as appropriate K-interpolation spaces with general weights and then applying the reiteration technique for K-interpolation spaces.

Samko's contribution to the theory and applications of Morrey and Morrey type spaces is well recognized. No doubt that the following three papers were under influ-

ence of and related to Samko's results. In "Weighted boundedness of certain sublinear operators in generalized Morrey spaces on quasi-metric measure spaces under the growth condition" by Natasha Samko, weighted boundedness of Calderón–Zygmund and maximal singular operators in generalized Morrey spaces on quasi-metric measure spaces, in general non-homogeneous for a certain class of weights only under the growth condition on the measure is proven. Weights and characteristic of the spaces are independent of each other. Weighted boundedness of the maximal operator is also proved in the case where lower and upper Ahlfors exponents coincide with each other. It is worth mentioning, that the approach used is based on two important ingredients. The first one is a certain transference theorem, where without use of homogeneity of the space, a condition which guarantees that every sublinear operator with the size condition bounded in Lebesgue space is also bounded in the generalized Morrey space is provided. The second one is a reduction theorem which reduces weighted boundedness of the considered sublinear operators to that of weighted Hardy operators and non-weighted boundedness of some special operators. In the paper "Compactness of commutators of integral operators with functions in Campanato spaces on Orlicz–Morrey spaces" by Satoshi Yamaguchi and Eiichi Nakai, the commutators $[b, T]$ and $[b, I_\rho]$ are considered, where T is a Calderón–Zygmund operator, I_ρ is a generalized fractional integral operator and b is a function related to generalized Campanato spaces. A necessary and sufficient condition for the compactness of $[b, T]$ and $[b, I_\rho]$ on Orlicz–Morrey spaces is given. Since the Orlicz–Morrey spaces unify Orlicz and Morrey spaces, while the Campanato spaces unify BMO and Lipschitz spaces, the obtained results contain many previous results as corollaries. Finally, in "Singular and fractional integral operators on weighted local Morrey spaces" by Javier Duoandikoetxea and Marcel Rosenthal, a characterization of the weighted inequalities for the Riesz transforms on weighted local Morrey spaces is obtained. The condition is sufficient for the boundedness on the same spaces of all Calderón–Zygmund operators suitably defined on the functions of the space. In the case of the fractional maximal operator and the fractional integral a characterization valid for exponents satisfying the Sobolev relation is given. For power weights, sharp results for these operators in the usual versions of weighted Morrey spaces, neither restricted to the Sobolev relation of the exponents nor to the one-weighted setting, are obtained.

One of the main interests of Stefan Samko in the last decades is the study of the properties and especially specific features of the spaces with variable exponents. Since such spaces are the analogues of the corresponding "regular" spaces, such works are closely related to the other topics in this collection. We begin with "Extrapolation and the Boundedness in Grand Variable Exponent Lebesgue Spaces Without Assuming the log-Hölder Continuity Condition, and Applications" by Vakhtang Kokilashvili and Alexander Meskhi. The boundedness of the Hardy–Littlewood maximal operator, and the weighted extrapolation in grand variable exponent Lebesgue spaces are established provided that the Hardy–Littlewood maximal operator is bounded in appropriate variable exponent Lebesgue space. Moreover, some bounds for the norm of the Hardy–Littlewood maximal operator in these spaces are given. As corollaries, appropriate norm inequalities and the boundedness of operators of Harmonic Analysis such as maximal and sharp maximal functions; Calderón–Zygmund singular integrals, commutators of singular integrals in grand variable exponent Lebesgue

spaces are established. Finally, applying the boundedness results of integral operators of Harmonic Analysis, the direct and inverse theorems on the approximation of 2π-periodic functions by trigonometric polynomials in the framework of grand variable exponent Lebesgue spaces are derived. Further, Alexandre Almeida in "Maximal operator in variable Stummel spaces" proved that variable exponent Morrey spaces are closely embedded between variable exponent Stummel spaces and showed that such embeddings are strict in all the cases under consideration by constructing counterexamples. As a consequence, continuous embeddings between generalized Morrey spaces and generalized Stummel spaces are established, as well as between Stummel classes (vanishing Stummel spaces). In particular, embeddings into a new Stummel class of functions with some vanishing property at infinity are obtained. Also a known result on the coincidence of Stummel spaces with a modification of Morrey spaces where the supremum norm is replaced by an integral L^p-norm is partially improved. The boundedness of the maximal operator in variable exponent Stummel spaces as well as in vanishing variable exponent Stummel spaces is also studied. Surprisingly, these boundedness results are new even for the constant exponent case.

Of course, weighted inequalities can by no means be forgotten in this context. Many sharp results in harmonic analysis are established by means of such inequalities, and sometimes can be achieved only in this way. The following two papers are nice examples of such a study for classical operators and classical spaces. In "Weighted inequalities for a superposition of the Copson operator and the Hardy operator" by Amiran Gogatishvili, Zdeněk Mihula, Luboš Pick, Hana Turčinová and Tuğçe Ünver, a three-weight inequality for the superposition of the Hardy operator and the Copson operator, namely,

$$\left(\int_a^b \left(\int_t^b \left(\int_a^s f(\tau)^p v(\tau)\,d\tau\right)^{\frac{q}{p}} u(s)\,ds\right)^{\frac{r}{q}} w(t)\,dt\right)^{\frac{1}{r}} \le C\int_a^b f(t)\,dt,$$

in which (a, b) is any nontrivial interval, q, r are positive real parameters and $p \in (0, 1]$, is studied. A simple change of variables can be used to obtain any weighted L^p-norm, with $p \ge 1$, on the right-hand side. Another simple change of variables can be used to equivalently turn this inequality into the one in which the Hardy and Copson operators swap their positions. We focus on characterizing those triples of weight functions (u, v, w) for which this inequality holds for all nonnegative measurable functions f with a constant independent of f. To this end, a new type of approach based on an innovative method of discretization, which enables avoiding duality techniques and therefore removing various restrictions that appear in earlier work, is used. As the title shows, "On the boundedness of the Hilbert transform from weighted Sobolev space to weighted Lebesgue space" by Vladimir D. Stepanov, deals with the eternal Hilbert operator. Under some restrictions on weight functions sufficient conditions for the boundedness of the Hilbert transform from weighted Sobolev space of the first order on the semi-axis to weighted Lebesgue space are obtained.

The appearance of the following work in this collection is justified not only by its level and interest but also that the study of certain differential operators was the subject of several joint publications of the author with Samko. Here, Vladimir Rabinovich in

"Dirac operators with delta-interactions on smooth hypersurfaces in $\mathbb{R}^n$", considers the Dirac operators with singular potentials $D_{\boldsymbol{A},\Phi,m,\Gamma\delta_\Sigma}$. Let $H^1(\Omega^\pm, \mathbb{C}^N)$ be the Sobolev spaces of N-dimensional vector-valued distributions $\boldsymbol{u}$ on $\Omega^\pm$, and

$$H^1(\mathbb{R}^n \setminus \Sigma, \mathbb{C}^N) = H^1(\Omega_+, \mathbb{C}^N) \oplus H^1(\Omega_-, \mathbb{C}^N).$$

The formal Dirac operator $D_{\boldsymbol{A},\Phi,m,\Gamma\delta_\Sigma}$ is associated with an unbounded in $L^2(\mathbb{R}^n, \mathbb{C}^N)$ operator $\mathcal{D}_{\boldsymbol{A},\Phi,m,\mathfrak{B}_\Sigma}$. The main goals of the paper are the study of self-adjointmess of the operators $\mathcal{D}_{\boldsymbol{A},\Phi,m,\mathfrak{B}_\Sigma}$ for uniformly regular C^2-hypersurfaces $\Sigma \subset \mathbb{R}^n$ and the essential spectra of $\mathcal{D}_{\boldsymbol{A},\Phi,m,\mathfrak{B}_\Sigma}$ for closed C^2-hypersurfaces $\Sigma \subset \mathbb{R}^n$.

One more popular subject in modern analysis is Poincaré type inequalities. In "Some non-standard biparametric Poincaré type inequalities through harmonic analysis", the author María Eugenia Cejas shows some non-standard Poincaré type estimates in the biparametric setting with appropriate weights. These results are derived by using variants from classical estimates exploiting the interplay between maximal functions and fractional integrals. Also, a sharper result is provided by using extrapolation techniques.

The idea of sparseness has proved to be of great importance and usefulness in modern harmonic analysis and approximation theory. The two following studies demonstrate the strength and usefulness of this notion. One of them, "Sparse non-smooth atomic decomposition of quasi-Banach lattices" by Naoya Hatano, Ryota Kawasumi and Yoshihiro Sawano, exibits a theory of non-smooth atomic decomposition for a large class of quasi-Banach lattices, including Morrey spaces, Lorentz spaces, mixed Lebesgue spaces as well as some related function spaces. As an application, an inequality comparing the fractional maximal operator and the fractional integral operator is considered. Some examples show that the restriction posed on quasi-Banach lattices are indispensable. This paper, which is a follow-up of the third author's earlier work, simplifies the proof of some existing results. The other paper, "Sparse grid approximation in weighted Wiener spaces" by Yurii Kolomoitsev, Tetiana Lomako and Sergey Tikhonov, treats approximation properties of multivariate periodic functions from weighted Wiener spaces by sparse grid methods constructed with the help of quasi-interpolation operators. The class of such operators includes classical interpolation and sampling operators, Kantorovich-type operators, scaling expansions associated with wavelet constructions, and others. The rate of convergence of the corresponding sparse grid methods in weighted Wiener norms as well as analogues of the Littlewood–Paley-type characterizations in terms of families of quasi-interpolation operators are obtained.

We are sure that the above overview of the whole collection draws an impressive picture of a variety of topics in harmonic analysis reflecting the interests or related to those of Stefan Samko. Hopefully, not only he will enjoy this broad collection but numerous readers will gain much from the presented results and methods.

3 Concluding Remarks

A conference dedicated to the 80th anniversary of Professor Samko was held in August, 23–26, 2021 (OTHA-2021) in the city of Rostov-on-Don in Southern Federal University, the alma-mater of Professor Stefan Samko. Majority of the authors of this issue took part at the conference, and the idea of the present issue is one of the results of the conference activities. There is another issue still, composed of ten research articles also dedicated to the 80th anniversary of Professor Samko. It will appear in the Journal of Mathematical Sciences (Series A).

The guest editors would like to express their deepest gratitude to the Editor-in-Chief of the Journal of Fourier Analysis and Applications, Professor Hans G. Feichtinger, for this excellent opportunity for forming this issue. It was a great experience for us as well as for our remarkable authors; we take this chance to warmly thank all of them for their valuable contribution.

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