# Research Programme

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## Research interests

*Analysis of PDEs and harmonic analysis*

- PDOs and ΨDOs on curved manifolds and microlocal analysis
- Commutative non-Euclidean Fourier analysis and its relation with quantum integrability
- Non-commutative harmonic analysis and global quantization on groups, homogeneous spaces and more general manifolds
- The correspondence between harmonic analysis and spectral theory
- Harmonic and spectral analysis on almost Abelian groups and homogeneous spaces

*Mathematical physics*

- QFT in curved spacetimes and mathematical cosmology
- Elementary particle states in homogeneous cosmological spacetimes
- Quantum integrability and quantization of non-linear systems

## Project description

The broad context of problems is analysis on almost Abelian groups and their homogeneous spaces. An almost Abelian group is a (connected) non-Abelian Lie group which possesses a codimension one Abelian normal subgroup. The most prominent representatives of this class are the 3-dimensional Heisenberg group $\mathbf{H}^3$, the group $ax + b$ of affine transformations on the line, and the isometry group E(2) of the Euclidean plane. Applications and interpretations of other almost Abelian Lie groups and algebras can be found here[1][2]. The class of almost Abelian groups is rather diverse and representative for solvable Lie groups from the group theory perspective, which makes it a very promising context for developing new methods in non-commutative harmonic analysis. The general objective is a comprehensive study of almost Abelian groups and homogeneous spaces with natural structures on them. This appears to be an ideal context for undergraduate research supervision in the following sense. Mathematics is learnt by doing it, but when you reach the more advanced topics, explicit examples become very rare and complicated. This often results in students having only an abstract understanding of things, without a hands-on command of details. Almost Abelian groups provide explicitly tractable (non-trivial, non-textbook-classical) examples of many advanced mathematical notions; covering manifolds, non-exonential groups, PDEs with variable coefficients, self-adjoint operators on non-compact domains etc. Students are very excited to see very real examples of what they know should exist, and contribute new results to mathematics on that way. Below are some of the particular research problems addressed in collaboration with undergraduate students.

[1] Z. Avetisyan, R. Verch. Explicit harmonic and spectral analysis in Bianchi I-VII-type cosmologies. *Class. Quant. Grav.*, 30(15), 2013

[2] Z. Avetisyan. Structure of almost abelian Lie algebras. *ArXiv:1610.05365*, 2016

- Classification up to isomorphism of all almost Abelian homogeneous spaces and an explicit description of their homotopy types.
- Explicit description of all discrete subgroups of a connected almost Abelian group, their classification up to isomorphism or automorphisms.
- Necessary and sufficient conditions for a connected almost Abelian group to admit a faithful matrix representation, and an explicit description of such a representation when it exists.

## Prerequisites

The present research project touches upon many advanced mathematical subjects, but no prior knowledge is required to participate in this programme except:

- Undergraduate level linear algebra (e.g., real and complex vector spaces and operators, operations with matrices)
- Undergraduate level analysis (e.g., differential and integral calculus of functions of several variables, vector fields)
- Strong problem solving skills
- Acquaintance with standards of mathematical writing (e.g., definitions, proofs)
- Modesty and ambition :-)

## Discussion

Of course, it is not expected that every student acquires a good understanding of all these subjects in the short time frame of eight weeks. There will be three main regimes of learning new material:

- Independent reading (starting well before the first day of the programme)
- Crash courses on specific topics during the programme
- Learn as you go (learning by doing)

The mathematical problems to be addressed are of highest scientific standards, and solving them will require much work and utmost devotion (and, obviously, talent). Students will be provided all the guidance needed, but will also be expected to be autonomous. The research output will be published in prestigious international journals and/or as part of a future monograph, and the participating students will act as coauthors.

## Working mode

This programme is intended to be run remotely. This assumes that the student has a high quality internet connection and a working computer capable of video conferencing (via Zoom) and with sufficiently large screen to conveniently read large amounts of technical text. The meeting time will be agreed upon depending on the student's time zone and schedule.