Research Program

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Research Tasks

In Mathematics, a composition operator C_φ is a linear operator defined by the rule $C_\varphi f = f \circ \varphi$ where φ denotes a selfmap of the unit disk. In physics, and specially the area of dynamical Systems, the composition operator is usually referred to as the Koopman operator; it is the left adjoint of the transfer operator. In Functional Analysis the domain of a composition operator is usually taken Banach spaces of analytic functions, or real functions.

Task 1. One of the most studied properties of composition operators is the compactness. The compactness problem in the space of Hardy H^2 was solved by Joel Shapiro 1987. Shapiro, not only solves the problem of compactness, in his paper he computed the exact distance of a composition operator to the ideal of compact operators, that is, its essential norm. Shapiro's article had a great impact and many researchers have characterized the compactness of a composition operators in different spaces of analytic functions. For example, Alexey Karapetyants and coworkers, characterized boundedness and weak compactness of composition operators defined on Holomorphic Holder type spaces. Our objective is to try to compute or estimate the essential norm of that composition operator and to derive some of the results by A. Karapetyants's result from these estimates.

Task 2. We recently characterized, on the Hardy space setting, the commutant of a composition operator C_φ when the symbol is a Moebius transformation mapping the unit disk into itself. Moreover, even for more general self maps of the unit disk. Solving some questions posed by C. Cowen, B. Cload, T.S. Worner, etc. twenty years ago. We are planning to continue those investigations for composition operators defined on the Hardy space, induced by other types of self maps of the unit disk.

Taks 3. A continuous operator T defined on a separable Fréchet space is said to be hypercyclic if the orbits of some vector x is dense. It is well known that hypercyclic properties of a given operator T propagate (in some specific way) to the commutant of the operator. We recently studied Hypercyclicity properties of operators that λ-commutes with the differentiation operator in the space of entire functions with the compact open topology. And we see that such hypercyclic properties are extensive also for many operators that commute up to a factor. We are interested in to know more about this property for some classical ocurring operators: Unilateral shifts, Toeplitz operators, composition operators, etc.

Prerequisites

Basic Functional Analysis and Operator Theory.
The ability to understand a research paper.

Working Mode

The main part of the work should be done by the student independently. The guidance will be provided. For online meetings we prefer G.meet and zoom.