

# INTERNATIONAL BIWEEKLY ONLINE SEMINAR ON ANALYSIS, DIFFERENTIAL EQUATIONS AND MATHEMATICAL PHYSICS

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Coordinators: Prof. Alexey Karapetyants, Prof. Vladislav Kravchenko

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**12 May 2022, 6 pm (UTC+3)**

**Semiclassical Approximation with Complex Phases for Constructing Effective  
Plancherel-Rotach type asymptotics of 1-D and 2-D orthogonal polynomials**

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Many orthogonal polynomials  $u(n, z)$ , ( $n$  is the number of the polynomial,  $z$  is its argument), for example, the Chebyshev, Hermite, Laguerre, Legendre, are determined by recurrent relations (or difference equations) of the second order. For large numbers of  $n$ , they are approximated by the exponent, trigonometric, or special functions of a complex argument. For example, Hermite polynomials are approximated by the Plancherel-Rotach formulas, in which the special function is the Airy function  $Ai$ , Legendre polynomials are approximated by the zero-order Bessel function, and so on. We discuss an approach [1] to finding asymptotics of this type that are uniform in the variable  $z$ , based on the transition from discrete equations to continuous pseudodifferential equations in the variable  $x = nh$ , for functions  $w(x, z)$ , ( $u(k, z) = w(kh, z)$ , where  $h \sim 0$  ( $1/n$ ) is an artificial small parameter) and the subsequent application of the semiclassical approximation with complex phases to them. The developed approach is generalized for 2-D Hermitian type orthogonal polynomials  $H(n_1, n_2, z, a)$  with two indices  $n_1, n_2$ . This part of the talk contains the results recently obtained together with A. I. Aptekarev and D. N. Tulyakov [2,3].

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The talk is based on joint work with Prof. Anna Tsvetkova.

## Bibliography

- [1] A. I. Aptekarev, S. Yu. Dobrokhotov, D. N. Tulyakov, A. V. Tsvetkova, Plancherel-Rotach type asymptotics for multiple orthogonal Hermite polynomials and recurrence relations, *Izvestiya: Mathematics* 86:1, 32-91
- [2] S. Yu. Dobrokhotov and A. V. Tsvetkova, An Approach to Finding the Asymptotics of Polynomials Given by Recurrence Relations, *Russian Journal of Mathematical Physics*, Vol. 28, No. 2, 2021, pp. 198-223
- [3] S. Yu. Dobrokhotov and A. V. Tsvetkova, Asymptotics of multiple orthogonal Hermite polynomials  $H(n_1, n_2, z, \alpha)$  determined by a third-order differential equation, *Russian Journal of Mathematical Physics*, Vol. 28, No. 4, 2021, pp. 439-454

\*Seminar website: <https://msrn.sfedu.ru/sl>. The seminar uses Microsoft Teams online platform.

Please send questions to [ademp.seminar@gmail.com](mailto:ademp.seminar@gmail.com) (Tatiana Andreeva, scientific secretary).

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