INTERNATIONAL BIWEEKLY ONLINE SEMINAR ON ANALYSIS, DIFFERENTIAL EQUATIONS AND MATHEMATICAL PHYSICS

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Fenton type minimax problems for sum of translates functions

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There are several antecedents of our study, one being the attack on Bary's Conjecture by P. Fenton in 2000. Fenton based his approach on the following minimax type lemma.

Theorem 1 (Fenton[1]). Let the "field function" $J:(0,1)\to\mathbb{R}$ be concave, and the "kernel function" $K:(-1,1)\to[-\infty,\infty)$ be monotone, strictly concave and C^2 both on $(0,1)$ and on $(-1,0)$, with $K''<0$ on $(-1,0)\cup(0,1)$ and satisfying the cusp condition

(1) $$\lim_{t\uparrow o} K'(t) = -\infty \quad and \quad \lim_{t\downarrow o} K'(t) = \infty.$$

Then for the so-called "sum of translates function" $F(y;t) := J(t) + \sum_{j=1}^{n} K(t-y_j)$ there exists an extremal (minimax) node system $w := (w_1,\dots,w_n)$ in the open simplex $S := \{x: 0 < x_1 < \cdots < x_n < 1\}$:

(2) $$M(\bar{S}) := \inf_{y\in\bar{S}} \overline{m}(y) := \inf_{y\in\bar{S}} \sup_{[0,1]} F(y;\cdot) = \overline{m}(w) := \sup_{[0,1]} F(w;\cdot).$$

Moreover, $F(w;\cdot)$ equioscillates on the intervals $L_j(w) = [w_j, w_{j+1}]$:

$$m_j(w) := \sup_{[w_j,w_{j+1}]} F(w,\cdot) = m_i(w) := \sup_{[w_i,w_{i+1}]} F(w,\cdot) \qquad (0 \le i,j \le n).$$

Furthermore, w is the unique equioscillation node system, and it is the only maximin point: $m(S) := \sup_{y\in\bar{S}} \underline{m}(y) = \underline{m}(w)$, too.

Bary's Conjecture was solved by Godldberg already before Fenton, but Fenton's approach found other applications and became a powerful tool of several investigations. In the lecture we will show how this lemma of Fenton can be generalized and used in various contexts in the constructive theory of functions. In particular, we will explain new findings even about the most classical Chebyshev problem on the minimal norm monic polynomial on the interval $[0,1]$.

[1] P. Fenton, A min-max theorem for sums of translates of a function, J. Math. Anal. Appl. 244 (2000), no. 1, 214-222.

*Seminar website: https://msrn.sfedu.ru/sl. The seminar uses Microsoft Teams online platform. Please send questions to ademp.seminar@gmail.com (Tatiana Andreeva, scientific secretary).

The seminar is organized by the coordinators Alexey Karapetyants and Vladislav Kravchenko within the activities of the Regional Mathematical Center of the Southern Federal University in collaboration with Institute of Mathematics, Mechanics and Computer Sciences of the Southern Federal University and the OTHA research group in Operator Theory and Harmonic Analysis.

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