INTERNATIONAL BIWEEKLY ONLINE SEMINAR ON ANALYSIS, DIFFERENTIAL EQUATIONS AND MATHEMATICAL PHYSICS

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Schrödinger equation with finitely many δ-interactions: closed form, integral and series representations for solutions

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In this talk, we discuss recent results concerning the one-dimensional Schrödinger equation with a finite number of δ-interactions:

$$L_{q,\Im_N} y := -y'' + \left(q(x) + \sum_{k=1}^{N} \alpha_k \, \delta(x - x_k) \right) y = \lambda y, \qquad 0 < x < b, \;\; \lambda \in \mathbb{C}.$$

We present a closed form solution expressed in terms of the solution of the unperturbed equation

$$L_q y := -y'' + q(x) y = \lambda y, \qquad 0 < x < b, \;\; \lambda \in \mathbb{C},$$

along with a corresponding transmutation (transformation) operator $T^f_{\Im_N}$, which is obtained in the form of a Volterra integral operator. With the aid of the spectral parameter power series method, a practical construction of the image of the transmutation operator on a dense set is presented, and it is proved that the operator $T^f_{\Im_N}$ transmutes the second derivative into the Schrödinger operator $L_{q,\Im_N}$ on a Sobolev space H^2. A Fourier-Legendre series representation for the integral transmutation kernel is developed, from which a new representation for the solutions and their derivatives, in the form of a Neumann series of Bessel functions (NSBF), is derived. We further explore the utility of this NSBF representation in solving direct and inverse spectral problems.

This talk is based on the joint work [1] with Vladislav V. Kravchenko (CINVESTAV, Queretaro).

1. V. V. Kravchenko, V. A. Vicente-Benitez, Schrödinger equation with finitely many δ-interactions: closed form, integral and series representations for the solutions, Anal. Math. Phys. 14, 97 (2024) https://doi.org/10.1007/s13324-024-00957-4.

*Seminar website: https://msrn.sfedu.ru/sl. The seminar uses Microsoft Teams online platform.
Please send questions to ademp.seminar@gmail.com (Tatiana Andreeva, scientific secretary).

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