ЮЖНЫЙ ФЕДЕРАЛЬНЫЙ УНИВЕРСИТЕТ
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Regional Mathematical Center
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International scientific online seminar on
Analysis, Differential Equations and Mathematical Physics

Coordinators: Prof. Alexey Karapetyants, Prof. Vladislav Kravchenko

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Dirac-type operators and applications

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Clifford analysis is a higher-dimensional analog of classical complex function theory and refinement of harmonic analysis. The core and center of the theory is the Dirac operator

$$D = \sum_{i=1}^{n} e_i \frac{\partial}{\partial x_i}, \quad \text{where} \quad e_i, \quad i = 1, ..., n,$$

are the generating elements of the Clifford algebra $\mathbb{R}_n$ and fulfill the non-commutitive multiplication rules

$$e_i e_j + e_j e_i = -2\delta_{ij}.$$

The Dirac operator consists of a radial component and a phase, where $|D| = \sqrt{-\Delta}$ is the radial Dirac operator or the square root of the negative Laplacian. $H = \sum_{j=1}^{n} e_j R_j$ is the Hilbert operator and $R_j, j = 1, \ldots, n$, are the Riesz operators with $\widehat{R_j f}(\underline{\xi}) = -i\frac{\xi_j}{|\underline{\xi}|}\hat{f}(\underline{\xi})$. Where $\hat{}$ denotes the classical Fourier transform in $\mathbb{R}^n$.

The zero solutions of the Dirac equation are called monogenic functions. Cauchy integrals can represent monogenic functions. To describe the boundary values of monogenic functions, the Hilbert operators and Hardy spaces are essential.

We will consider several generalizations of Dirac operators and Hilbert transformations and their applications in optics. An essential tool in these considerations will be the Fourier symbol of these operators and multiplier theorems. Specifically, we will consider Dirac-type operators $D_{\mathcal{H}} = |D|\mathcal{H}$, where the Hilbert transform is replaced by an arbitrary pseudo-differential operator $\mathcal{H}$ of degree zero. We call the zero solutions of the associated Dirac operator quasi-monogenic functions. We will consider an example of such an operator and its application in optics.

Fractional Dirac and Hilbert operators represent another type of modification. Fractional Hilbert operators H^α have applications in optics, and we will discuss this application. We consider the Cauchy problem for fractional Dirac operators $D^{\alpha,\theta} = (\sqrt{-\Delta})^\theta H^\alpha$ and the associated semigroups. Depending on the choice of parameters of the fractional Dirac operator, classical weighted spaces, such as Sobolev spaces or modulation spaces, and exotic spaces such as Beurling spaces (or generalized Sobolev spaces) are suitable to describe the mapping properties.

*Seminar website: https://rmc.sfedu.ru/seminar. The seminar uses Microsoft Teams online platform.
Please send questions to pichugina@sfedu.ru (Olga Pichugina, scientific secretary).

The seminar is organized by the Regional Mathematical Center of the Southern Federal University in collaboration with Institute of Mathematics, Mechanics and Computer Sciences of the Southern Federal University and the special Interest ISAAC-OTHA group in Operator Theory and Harmonic Analysis.

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