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A Sturm-Liouville equation on the crossroads of discrete and continuous hypercomplex analysis

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The subject of our talk is a different type of complexification as usual for the treatment of generalized Cauchy-Riemann equations by methods of hypercomplex analysis. It led us to a special class of multivariate polynomials with coefficients which, in the case of two real variables, are identical with Vietoris numbers. In 1958 Vietoris encountered this number sequence in connection with the positivity of trigonometric sums, which are also relevant in the theory of special functions. Through a recurrence relation they lead us to a Sturm-Liouville equation via the calculus of holonomic differential equations. Consequently, one of the particular solutions of this equation serves as generating function for those numbers.

The problem of complexification in hypercomplex analysis has the following historical background. About 50 years ago, E. M. Stein and G. Weiss proved in their seminal paper [1] the “correspondence of irreducible representations of several rotation groups to first order constant coefficient partial differential equations generalizing the Cauchy-Riemann equations”.

They showed how certain properties of complex one-dimensional function theory extend to solutions of those systems of PDE. The list of systems includes the generalized Riesz system, the Moisil-Theodoresco system, spinor systems as n-dimensional generalization of Diracs equations, Hodge - de Rham equations, etc. Their motivation for proving that correspondence between represent-tation groups and partial differential equations were merely of qualitative nature and deeply connected with properties of harmonic functions in several real variables.

Around the same time the renewed interest in quaternions and their embedding in Clifford Algebras together with deep relations to symmetry groups provoked a fast-growing number of papers by physicists working in Quantum Mechanics and Quantum-Field Theory [2]. Decades later, mathematicians successfully developed (or renewed from the 30ies) analytical tools for the treatment of all kinds of generalized Cauchy-Riemann or Dirac equations, in the beginning often influenced by [3], [4]. Naturally, this type of generalized function theory heavily relied on representation theoretic and algebraic tools, functional analytic and topological principals, etc., but less on instruments or results from classical complex function theory. The results in [3] partially contributed to that by suggesting that only Riemann’s approach via conjugate harmonic functions (like it was the case in [1]) were a meaningful approach to Quaternionic analysis via the usual choice of quaternionization (see V. I. Arnold’s philosophy in [5]). But, as we will see, the use of several hypercomplex variables, showing that hypercomplex analysis can also be considered as function theory in co-dimension one (see [6]), opened the eyes to new insights.

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[4] F. Brackx, R. Delanghe, and F. Sommen: Clifford analysis, Research Notes in Mathematics, Vol. 76, Pitman, Boston, 1982, 308 pp
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