



International scientific online seminar on  
 Analysis, Differential Equations and Mathematical Physics

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Fredholm property and essential spectrum of 3-D Dirac operators  
 with regular and singular potentials

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We consider the 3 – D Dirac operator

$$\mathbb{D}_{\mathbf{A},\Phi,Q_{\text{sin}}} u(x) = (\mathbb{D}_{\mathbf{A},\Phi} + Q_{\text{sin}}) u(x), x \in \mathbb{R}^3$$

with variable regular magnetic and electrostatic potentials, and with singular potentials  $Q_{\text{sin}}$  on  $\mathbb{R}^3$  where

$$\mathbb{D}_{\mathbf{A},\Phi} = \sum_{j=1}^3 \alpha_j (i\partial_{x_j} + A_j(x)) + \alpha_0 m + \Phi(x)I_4$$

is the 3 – D Dirac operator,  $Q_{\text{sin}} = \Gamma\delta_{\Sigma}$  is the singular potential where  $\delta_{\Sigma}$  is the delta-function with support on a hypersurface  $\Sigma \subset \mathbb{R}^3$  which divides  $\mathbb{R}^3$  on two open domains  $\Omega_{\pm}$  with the common boundary  $\Sigma$ ,  $\Gamma(s) = (\Gamma_{ij}(s))_{i,j=1}^4$  is a  $4 \times 4$  matrix-function on  $\Sigma$ ,  $u$  is a vector-function on  $\mathbb{R}^3$  with values in  $\mathbb{C}^4$ ,  $\alpha_j, j = 0, 1, 2, 3$  are the standard  $4 \times 4$  Dirac matrices,  $I_4$  is the  $4 \times 4$  unit matrix. We associate with the formal Dirac operator  $\mathbb{D}_{\mathbf{A},\Phi,Q_{\text{sin}}}$  an unbounded operator  $\mathcal{D}$  in  $L^2(\mathbb{R}^3, \mathbb{C}^4)$  generated by  $\mathbb{D}_{\mathbf{A},\Phi}$  with domain in  $H^1(\Omega_+, \mathbb{C}^4) \oplus H^1(\Omega_-, \mathbb{C}^4)$  consisting of functions satisfying interaction (transmission) conditions on  $\Sigma$ . We consider the self-adjointness of operator  $\mathcal{D}$ , its Fredholm properties, and the essential spectrum for the cases if  $\Sigma$  is either a closed  $C^2$ –hypersurface or an unbounded  $C^2$ –hypersurface with a regular behavior at infinity. We give applications to 3D–Dirac operators with singular potentials describing the electrostatic and Lorentz scalar  $\delta$ –shell interactions.

\*Seminar website: <https://rmc.sfedu.ru/seminar> . The seminar uses Microsoft Teams online platform. To join the seminar, please send a request to [pichugina@sfedu.ru](mailto:pichugina@sfedu.ru) (Olga Pichugina, scientific secretary).

The Seminar is organized by the Regional Mathematical Center of the Southern Federal University in collaboration with Institute of Mathematics, Mechanics and Computer Sciences of the Southern Federal University and the special Interest ISAAC-OTHA group in Operator Theory and Harmonic Analysis.

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